Papers

A more accurate formula for calculating the net longwave radiation flux in the Baltic Sea

OCEANOLOGIA, 49 (4), 2007.
pp. 449–470.

KEYWORDS
Longwave radiation
Baltic Sea

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Received 1 August 2007, revised 5 November 2007, accepted 9 November 2007.

Abstract

A new, more accurate formula for calculating the net longwave radiation flux $LW\uparrow\downarrow$ has been devised for the Baltic Sea region. To this end, the following sets of simultaneously measured data regarding the longwave radiation of the sea and the atmosphere were used: the temperatures of the sea surface and its contiguous air layer, the water vapour pressure in the air above the water, and the cloud cover. These data were gathered during numerous research cruises in the Baltic in 2000–03 and were supplemented by satellite data from Karlsson (2001) characterising the cloud cover over the whole Baltic. The formula established for $LW\uparrow\downarrow$ can be written in the form of three alternative equations, differing with respect to their cloud cover functions:

$$LW\uparrow\downarrow = 0.985\sigma T_s^4 - \sigma T_a^4(0.685+0.00452e)\begin{cases}(1+dn^2) & \text{average for all cloud types} \quad \text{(Z1)}\\ (1+d_i n^2) & \text{separately for low-, mid- and high-level clouds} \quad \text{(Z2)}\\ (1+d_i n^{\gamma_i}) & \text{separately for low-, mid- and high-level clouds} \quad \text{(Z3)}\end{cases}$$

The complete text of the paper is available at http://www.iopan.gda.pl/oceanologia/

where σ – Stefan-Boltzmann constant; T_s – sea surface temperature [K]; T_a – air temperature [K]; e – water vapour pressure [mbar]; n – total cloud amount $[0-1]$; d – mean empirical dimensionless coefficient, determined for all cloud types or for particular months (see Tables 3 and 4); d_i – empirical coefficient determined for the quadratic function: $d_1 = 0.39$ for low-level clouds, $d_2 = 0.305$ for mid-level clouds, $d_3 = 0.22$ for high-level clouds; d_i – empirical coefficient determined as follows: $d_1 = 0.39$ for low-level clouds when $\gamma_1 = 1.3$, $d_2 = 0.29$ for mid-level clouds when $\gamma_2 = 1.1$; $d_3 = 0.17$ for high-level clouds when $\gamma_3 = 0.96$. The improved accuracy of this formula (RMSE $\cong$ 10 W m^{-2}) is due chiefly to the establishment of functions and coefficients characterising the cloud cover over the Baltic in particular months of the year and their incorporation into it.

1. Introduction

Most of the mathematical formulas describing the net longwave radiation flux at the sea surface $LW\uparrow\downarrow$ make use of the dependence of this flux on simple meteorological parameters such as the sea surface temperature T_s, air temperature T_a, water vapour pressure e or dew point T_{Dew}, and the total cloud amount n (Fung et al. 1984, Dera 1992, Josey et al. 2003). Empirical verification of these formulas, as carried out by their authors, shows that the values of $LW\uparrow\downarrow$ calculated on the basis of the same meteorological data that they used for deriving their formulas in a given marine or terrestrial region are encumbered with a Root Mean Square Error (RMSE) of 14–20 W m^{-2} (Bignami et al. 1995, Zapadka et al. 2001, Josey et al. 2003). However, if an empirical dataset is applied to formulas other than the one derived from that particular dataset, this error may be much greater. A number of authors investigating this problem have tested various formulas on data from the regions for which they produced their own formulas (some of these are listed in Table 1). Bignami et al. (1995) performed their studies in the Mediterranean region and established a formula which we designate here as B95. Among other formulas, they tested the expression obtained by Clark et al. (1974) (C74). It is a modified version of Berliand & Berliand's (1952) formula (BB52), derived for terrestrial areas. Bignami et al. (1995) showed that values of $LW\uparrow\downarrow$, obtained on the basis of C74 and meteorological data from the Mediterranean Sea, have RMSE = 25 W m^{-2} and a mean bias error (MBE) = 21 W m^{-2}. Josey et al. (1997) also tested C74 using meteorological data from the SW Atlantic and obtained the downward component of the flux $LW\downarrow$ with RMSE = c. 16 W m^{-2}. In another study, Josey et al. (2003) used empirical data from a cruise in the Atlantic along the 20°W meridian from 20°N to 63°N; the values of $LW\downarrow$ they calculated with C74 have RMSE = 18 W m^{-2} and MBE = 11.7 W m^{-2}. They laid stress on the fact that the magnitudes of these errors were linked to cloud type and the range of variation of the empirical material. They derived two formulas

Table 1. Some existing formulas for calculating the net longwave radiation flux at the sea surface $LW\uparrow\downarrow$

Symbol	Formula	Reference
BB52	$\varepsilon\sigma T_a^4(0.39 - 0.05e^{1/2})(1 - Cn) + 4\varepsilon\sigma(T_s - T_a)^*$	Berliand & Berliand (1952)
C74	$\varepsilon\sigma T_s^4(0.39 - 0.05e^{1/2})(1 - \lambda n^2) + 4\varepsilon\sigma T_s^3(T_s - T_a)^{**}$	Clark et al. (1974)
B95	$\varepsilon\sigma T_s^4 - (\sigma T_a^4(0.653 - 0.00535e))(1 + 0.1762n^2)$	Bignami et al. (1995)
J03a	$\varepsilon\sigma T_s^4 - \sigma(T_a + 10.77n^2 + 2.34n - 18.44)^4$	Josey et al. (2003)
J03b	$\varepsilon\sigma T_s^4 - (1 - \alpha_L)$ $\sigma(T_a + 10.8n^2 + 2.3n - 18.4 + 0.84(D + 4.01))^{4***}$	Josey et al. (2003)
	where $D = T_{Dew} - T_a$; $T_{Dew} = 34.07 + 4157/\ln(2.1718 \times 10^8/e)$	
Z01	$\varepsilon\sigma T_s^4 - \sigma T_a^4(0.732(1 - \exp(-0.476e)(1 - 0.067n + 0.301n^2)))$	Zapadka et al. (2001)

$^*\varepsilon$ is the emissivity of the water surface.
C is a cloud cover coefficient dependent on latitude, season and cloud base altitude (Berliand & Berliand 1952).
$^{**}\lambda$ is a latitude-dependent cloud cover coefficient, taken to be 0.75.
$^{***}\alpha_L$ is the longwave reflectivity.

of their own (J03a and J03b – see Table 1), which they tested using the same meteorological data. The values of just the downward component $LW\downarrow$ obtained on the basis of these formulas have the following RMSE: J03a = 14.7 W m^{-2}; J03b = 11.6 W m^{-2}. In contrast, Zapadka et al. (2001) applied formula B95 to Baltic Sea data and obtained RMSE = 21 W m^{-2} and MBE = 14 W m^{-2}. RMSE for the formula of Zapadka et al. (2001, see Z01 in Table 1) is 19 W m^{-2}, which in comparison with other results is relatively large. Josey et al. (2003) also tested the formula B95, obtaining MBE = 12.1 W m^{-2} and RMSE = 20.8 W m^{-2}. In Bignami et al. (1995) these errors were 0.3 W m^{-2} and 13.7 W m^{-2} respectively. Such large discrepancies in the above MBE and RMSE are probably due to the local character of the various formulas. The cloud cover turns out to have the greatest effect on the net flux $LW\uparrow\downarrow$. In the formulas, the cloud factor is accounted for by the introduction of a cloud cover function. Various authors have shown that the emissivity of different cloud types can vary: from c. 0.13 for cirrus clouds to c. 1 for stratus and stratocumulus clouds (Allen 1971, Murcray et al. 1974; see also Paltridge & Platt 1976). These very different emissivities and the altitudes of the various cloud types directly affect the value of the downward longwave radiation flux $LW\downarrow$ reaching the sea surface. Therefore, the application of formulas containing only a total cloud amount parameter to spatio-temporal analyses of the net radiation flux $LW\uparrow\downarrow$ may give rise to considerable errors, since an empirical coefficient as a function of cloud cover may not adequately reflect a future situation investigated at a different time and place. This is because, given similar thermal conditions and the same total cloud amount, the values of $LW\uparrow\downarrow$ measured under, e.g., cirrus and cumulus clouds, are completely different. Indeed, our preliminary measurements indicate that the values obtained in these two situations can differ by as much as 90 W m^{-2}. The use in formulas of additional cloud cover data, such as the cloud base height, and combining this with data on the spatio-temporal distribution of the clouds over the sea area under investigation would undoubtedly improve the reliability of $LW\uparrow\downarrow$ estimates. A formula incorporating such information could also be very effectively used in satellite algorithms.

The aim of the present study is therefore to improve the formula presented in Zapadka et al. (2001) by incorporating into it information on cloud base height and modifying the empirical coefficients and functional relationships appearing in it. The empirical material used to this end is far more wide-ranging than in Zapadka et al. (2001), and the measurement methodologies are equivalent to those applied by Bignami et al. (1995) and Josey et al. (2003). The empirical formulas devised so far for marine areas have not included any information on cloud base height. Now, it is true that Berliand & Berliand's (1952) formula does contain this information, but it applies to terrestrial, not marine, areas. We compare the values of $LW\uparrow\downarrow$ for the Baltic, calculated with the aid of our new formula, with those evaluated using formulas B95 and J03, and we attempt to interpret the reasons for the discrepancies between the results.

The formula worked out in this paper may be successfully applied to the estimation of spatio-temporal distributions of the net longwave radiation flux $LW\uparrow\downarrow$ over the Baltic. A further objective of this work was therefore to find the mean annual and monthly values of an empirical coefficient characterising the cloud types occurring in the Baltic Sea region. This was achieved with the aid of long-term satellite observations of cloud types over the Baltic by Karlsson (2001).

2. Empirical material

The empirical material for the present work was obtained during cruises of r/v 'Oceania' in the southern Baltic and Baltic Proper in the spring (April, May), summer (June, August), autumn (September, October, November) and winter (February) seasons of 2000–03. The database thus contains information gathered under practically all the weather conditions likely to be experienced in the Baltic during the year. It includes the following parameters: sea surface temperature, air temperature, air humidity, cloud cover, classification of clouds with respect to base level, downward longwave radiation flux and upward longwave radiation flux. All the measured magnitudes used in the analysis were averaged over 10-minute time intervals around the time of the cloud observations. The computer-recorded parameters were sampled with a frequency of 1 second; file-saved values were the means for 1 minute.

SST was measured in situ with an electronic sensor connected to a computer. Measurements were made both when the ship was moving and when it was at anchor. The semi-conductor temperature sensor and its electrical lead were woven into the end of a rope supported on floats just below the water surface. The end of the rope was free, so as to allow the sensor to float freely, slightly immersed in the water (with the ship both at anchor and drifting). When the ship was moving, the rope and sensor were hauled along from a long extension arm protruding from the ship's bows, thus enabling the temperature sensor to be immersed about 1.5 m in front of the bows. The layer of water flowing over the sensor will have been from a few millimetres to a few centimetres thick. Measurements were performed 24 hours a day. The in situ measurements were accurate to 0.1°C.

The air temperature was measured in two ways. The main technique employed an electronic sensor much like the one used to measure the sea surface temperature; this was placed in a special housing c. 5 metres above the water surface. Measurements were made over the same time intervals as SST. Occasionally, the air temperature was measured with an Assman psychrometer; this was done at selected hours in order to check the working of the electronic sensor. As in the case of the water temperature, the in situ measurements of air temperature were accurate to c. 0.1°C.

The relative humidity of the air was also measured in two ways. It was measured continuously with an electronic hygrometer situated c. 5 metres above the water surface (as in the case of the air temperature) and connected to a computer. This parameter was obtained as a percentage, which was then converted to water vapour pressure expressed in millibars. A calibrated Assman psychrometer was also used to measure the absolute humidity: as with the air temperature, it served to check the accuracy of the electronic hygrometer.

The total cloud amount and the type of clouds were assessed by direct observation of the sky or by analysing photographs of the sky taken from different angles with a digital camera. The total cloud amount was defined in oktas and then converted to a 0–1 scale. The cloud levels (low, mid or high) were determined by the type of clouds according to the international cloud classification (see The International Cloud Atlas). If, during the observations the levels of the clouds varied, it was assumed that the main source of the radiation emission towards the sea was the lowest level of clouds. Ambiguous cases were ignored. The clouds occurring during rainy weather were classified as low-level. The sky was treated as cloudless $n = 0$ when there were no clouds at all or they were located such that they could not influence the emission of radiation reaching the sensor on the ship (e.g. clouds on the horizon).

The downward and upward longwave radiation fluxes were measured using two CG1 pyrgeometers (Kipp & Zonen) with P-100 temperature sensors inserted in each. Both CG1 sensors (one facing the sky, the other facing the sea) were situated on a 3-metre extension arm in front of the ship's bows c. 3 m above the sea surface. This placing of the sensors was intended to minimise the effect of radiation from the ship itself on the readings. The whole arrangement was connected to a computer continuously recording the downward and upward longwave radiation, in the same way as the air and water temperatures. Table 2 gives the cruise dates and the number of data sets gathered for analysis. It also reports the number of clear and cloudy sky observations. The data sets were prepared for analysis by averaging them over ten-minute time intervals. Since the cruises took place at different times of the year, the measured parameters exhibit

Table 2. Dates of cruises in the Baltic and the number of data sets* collected in this work during these cruises for cloudless skies $n = 0$ and cloudy skies $n \neq 0$ (for high-, mid- and low-level clouds)

Date	Data set				
	$n = 0$	$n \neq 0$	high	middle	low
15–30 September 2000	52	70	27	43	–
18–22 October 2000	59	30	17	13	–
03–12 November 2000	–	58	1	21	36
16–25 February 2001	5	38	9	7	22
04–16 May 2001	32	49	21	10	18
05–19 September 2001	–	84	1	9	74
05–15 February 2002	13	118	17	20	81
April 2002	21	86	11	13	62
03–15 May 2002	17	21	3	15	3
11 June 2002	9	2	1	–	1
14–16 August 2002	19	–	–	–	–
02–12 February 2003	17	168	14	43	111
Total	**244**	**724**	**122**	**194**	**408**

* One data set denotes 10-minute mean values of parameters registered by the computer around the sky observation time.

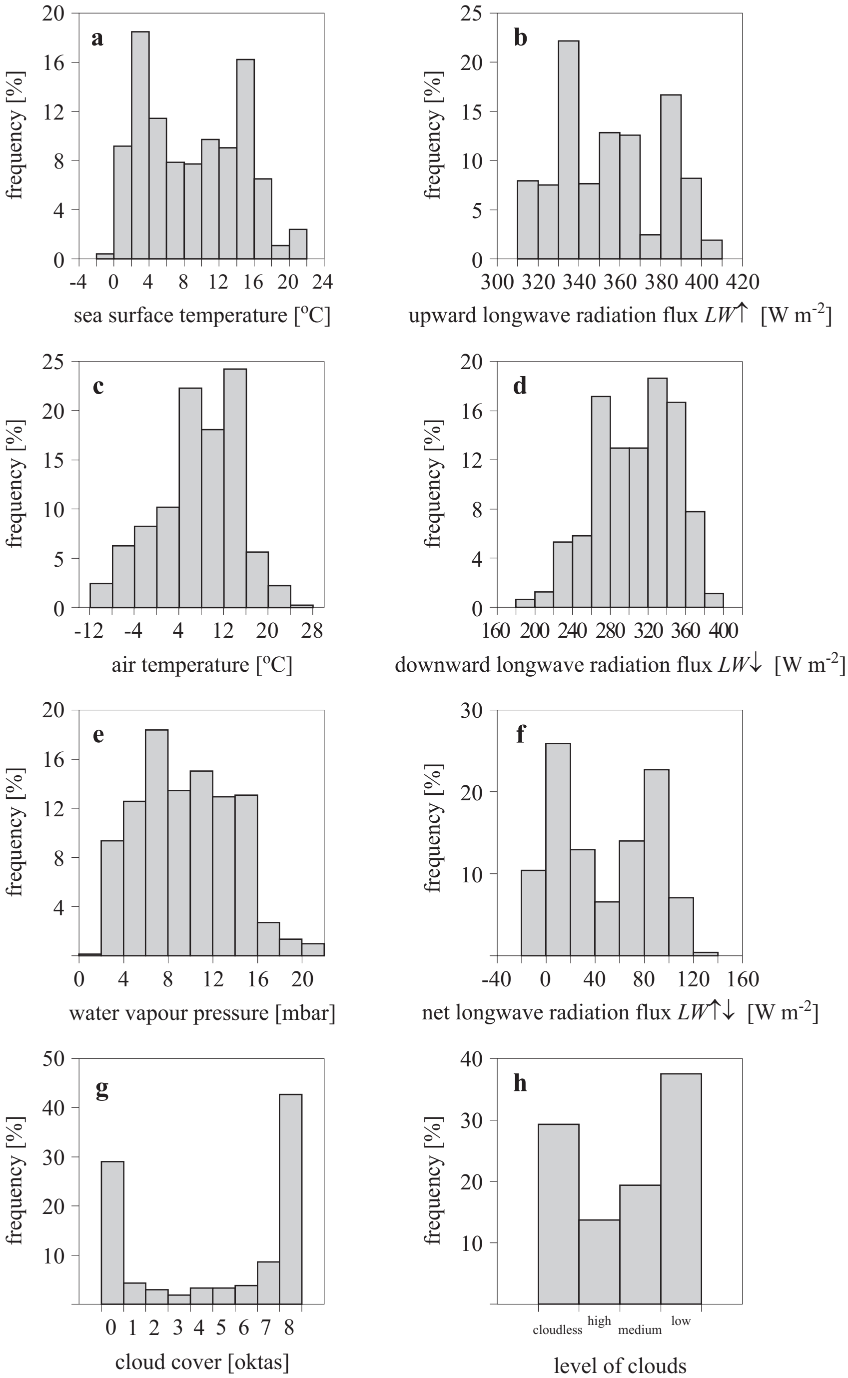

Figure 1. Characteristics of the empirical data gathered in the southern Baltic and Baltic Proper: sea surface temperature (a); upward longwave radiation (b); air temperature (c); downward longwave radiation (d); water vapour pressure (e); net longwave radiation (f); cloud cover (g); cloud level (h)

a wide range of variation (see Figure 1): water temperature – 0 – 20°C; air temperature – c. −14 – 26°C; water vapour pressure – 2 – 21 mbar; $LW\downarrow$ – c. 180 – c. 400 W m^{-2}; $LW\uparrow$ – 310 W m^{-2} – c. 400 W m^{-2}; $LW\uparrow\downarrow$ – c. −20 – 120 W m^{-2}.

3. The new formula

By definition, the net longwave radiation flux $LW\uparrow\downarrow$ is the difference between the upward longwave radiation flux emerging from the sea $LW\uparrow$ and the downward longwave radiation flux descending from the atmosphere $LW\downarrow$. It can be described with the aid of the Stefan-Boltzmann law, which relates the emissive power of an ideal black body at the absolute temperature T, supplemented by additional coefficients to allow for the fact that neither the sea nor the atmosphere are such black bodies:

$$LW\uparrow\downarrow = LW\uparrow - LW\downarrow = \varepsilon\sigma T_s^4 - \sigma T_a^4 g(e) f(n), \tag{1}$$

where $\sigma = 5.67 \times 10^{-8}$ W K^{-4}m^{-2} – Stefan-Boltzmann constant, T_s – sea surface temperature [K], T_a – temperature of the air just above the sea surface [K], e – water vapour pressure in the air [mbar], n – total cloud amount, ε – sea's emissivity (a dimensionless coefficient), $g(e)$ and $f(n)$ – dimensionless functions describing the emissivity of the atmosphere in relation to the water vapour pressure e in the air over the sea surface and to the cloud cover n. The prime objective of the present work was therefore to find the functions $g(e)$ and $f(n)$, and the coefficient ε, which enable eq. (1) to provide the most accurate possible calculations of the net longwave radiation flux of the Baltic Sea.

3.1. The upward longwave radiation flux $LW\uparrow$

The first step was to examine the relationship between the upward longwave radiation flux $LW\uparrow$ and the sea surface temperature T_s. It was assumed that the sea radiates like a grey body, where the degree of greyness corresponds to the emissivity ε. The real value of ε depends on the wavelength of the radiation (Masuda et al. 1988) and on the nature of the emitting surface. For the sea, ε is generally assumed to be 0.98 (Mikhaylov & Zolotarev 1970, Katsaros 1990). The authors of the research into net longwave radiation adopted different values of ε . In the formulas presented in Table 1 (B95, Z01, J03) $\varepsilon = 0.98$, and in formula BB52 $\varepsilon = 0.90$. In Bunker's 1976 study, in which formula C74 was adopted, $\varepsilon = 0.95$.

Such a value of ε is associated with the temperature of the water layer directly participating in the emission of longwave radiation from the sea to the atmosphere. However, the traditional means of measuring T_s frequently do not reflect the actual temperature of this layer. As various authors

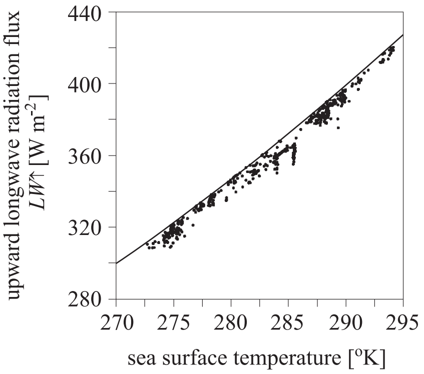

Figure 2. Empirical relationship of the upward radiation flux $LW\uparrow$ and surface water temperature in the Baltic Sea. The solid line was calculated from the Stefan-Boltzmann law

have shown, large temperature gradients can exist within it, which depend, among other things, on the wind speed and the insolation of the sea surface (Kent et al. 1996, Donlon et al. 2001). In the present work, therefore, a mean empirical coefficient ε was determined in order to link the upward flux $LW\uparrow$ with the surface temperature T_s measured as described in Section 2. With this approach, the coefficient ε, as determined here, can be used to estimate the upward flux $LW\uparrow$ from satellite measurements of T_s. Figure 2 illustrates the relationship between our measured $LW\uparrow$ values and the SST T_s. On this basis, the mean value of ε for the Baltic was established at $\varepsilon = 0.985$, with RMSE = 3.5 W m^{-2} and MBE = –0.1 W m^{-2}. The upward longwave radiation flux $LW\uparrow$ in the Baltic therefore takes the form

$$LW\uparrow = 0.985\sigma T_s^4. \tag{2}$$

3.2. The downward longwave radiation flux $LW\downarrow$

In most empirical formulas for the downward longwave radiation flux, $LW\downarrow$ is described as a function of the air temperature just above the sea surface, the water vapour pressure and the cloud cover. In the case of a cloudless sky, these formulas differ principally in the type of function associated with the water vapour pressure e and in the empirical coefficients they contain.

Let us define the emissivity of a cloudless atmosphere ε_a as the ratio of the downward longwave radiation flux in a cloudless atmosphere $LW\downarrow_{0,\,real}$ to the downward flux of such an atmosphere $LW\downarrow_{0,\,black} = \sigma T_a^4$ if it radiated as an ideal black body at temperature T_a:

$$\frac{LW\downarrow_{0,\,real}}{\sigma T_a^4} = \varepsilon_a = f(e). \tag{3}$$

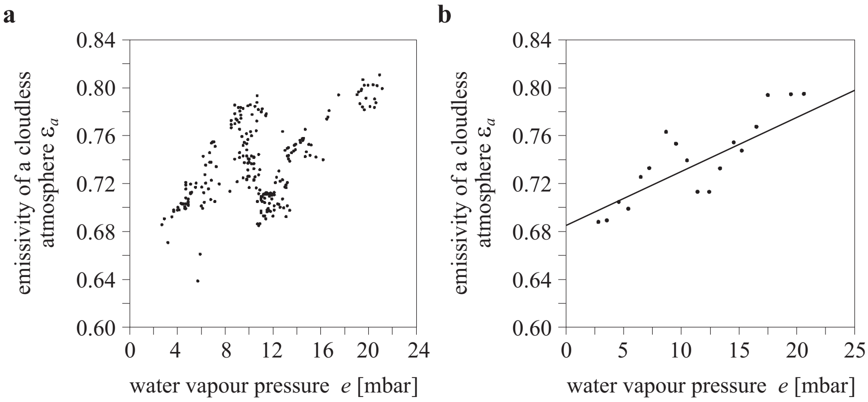

Figure 3. Empirical dependence of the emissivity of a cloudless atmosphere ε_a (see eqs. (3) and (4)) on the water vapour pressure e (a) and on the averaged values in the different intervals of the water vapour pressure e (b)

Then, by establishing a functional link between ε_a and e, we obtain a formula for the downward longwave radiation flux for a cloudless atmosphere $LW\downarrow_{0,\,real}$. If the ratio $\frac{LW\downarrow_{0,\,real}}{\sigma T_a^4}$ in expression (3) were constant, we would be dealing with an invariably grey body. But, as Figure 3 shows, the emissivity of a cloudless atmosphere increases with water vapour pressure. In order to approximate this empirical relationship with an analytical expression, approximating functions in the following two forms were tested: (a) $LW\downarrow_0 = \sigma T_a^4(a + be^{\chi})$ and (b) $LW\downarrow_0 = \sigma T_a^4(1 - a\exp(-(be)^{\gamma}))$. Functions of this kind have been used to calculate the radiation budgets of selected terrestrial and marine regions using terrestrial data (see e.g. Brunt 1932, Anderson 1952, Idso & Jackson 1969, Prata 1996, Malek 1997). Function (a) was tested for $\chi = 0.5 - 2$. The choice of function was determined by statistical analysis. Figure 3 also shows the dependence of mean $\varepsilon_a = \frac{LW\downarrow_{0,\,real}}{\sigma T_a^4}$ on the mean water vapour pressures e from intervals $\Delta e = 1$ mbar. It turns out that the best approximation of this dependence is given by the linear function

$$LW\downarrow_0 = \sigma T_a^4(a + be), \tag{4}$$

where the constants a and b, determined by regression methods, are respectively equal to 0.685 [dimensionless] and 0.00452 mbar^{-1}. Figure 4 compares measured values of the downward longwave radiation flux for a cloudless atmosphere $LW\downarrow_{0,\,real}$ with the values of the flux $LW\downarrow_{0,\,model}$ calculated from eq. (4). It also shows an error histogram of the estimated magnitude: MBE = 0.2 W m^{-2}, RMSE = 10 W m^{-2}. Using a linear function, Bignami et al. (1995) obtained similar estimates of the flux $LW\downarrow$ for cloudless skies over the Mediterranean Sea, with a comparable RMSE. They estimated

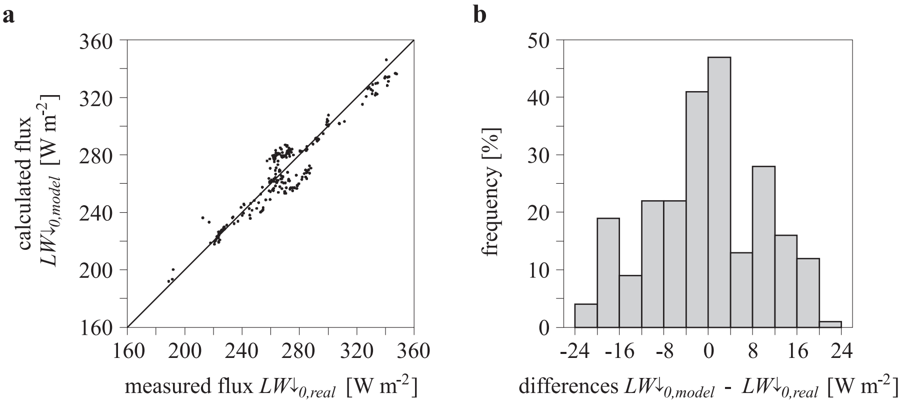

Figure 4. Comparison of measured values of the downward longwave radiation flux from a cloudless atmosphere $LW\downarrow_{0,\,real}$ with values of $LW\downarrow_{0,\,model}$ calculated according to eq. (4) (a), and the histogram of the differences between these two values (b)

constants a and b at 0.684 and 0.0056 mbar^{-1} respectively. These magnitudes are very much the same as those we determined for the Baltic; the slight differences may be due to the different vertical profiles of the humidity and air temperature over the Mediterranean and the Baltic. A further aspect is the different range of variation of the data analysed by Bignami et al. (1995). In their paper, the water vapour pressure varied from 9 to 25 millibars, whereas over the Baltic it is often lower. Statistical analysis of the downward component of B95 using Baltic data yielded an RMSE = 10.7 W m^{-2}, which is comparable to that using eq. (4), but a much larger MBE = 4 W m^{-2}.

The next step was to take into account the effect of clouds on the value of the downward flux $LW\downarrow$. Figure 5 illustrates the dependence of the ratio of the downward longwave radiation flux from a cloudy atmosphere to the corresponding flux from a cloudless atmosphere $\frac{LW\downarrow}{LW\downarrow_0}$ on the coefficient describing the cloud cover n on a $0-1$ scale for all weather situations recorded over the Baltic. If $n = 0$, the sky is cloudless, but if $n = 1$, it is totally overcast. $LW\downarrow$ is the downward longwave radiation flux measured for all states of the atmosphere, whereas $LW\downarrow_0$ is the corresponding flux for a cloudless atmosphere, calculated using eq. (4).

The relationship presented in Figure 5 was approximated by the function $f(n)$ in the form $1 + dn^{\gamma}$. Coefficients d and γ were determined by non-linear regression. It turns out that the best approximation of this relation is given by the quadratic function:

$$LW\downarrow = LW\downarrow_0 f(n) = \sigma T_a^4(0.685 + 0.00452e)(1 + dn^2), \tag{5}$$

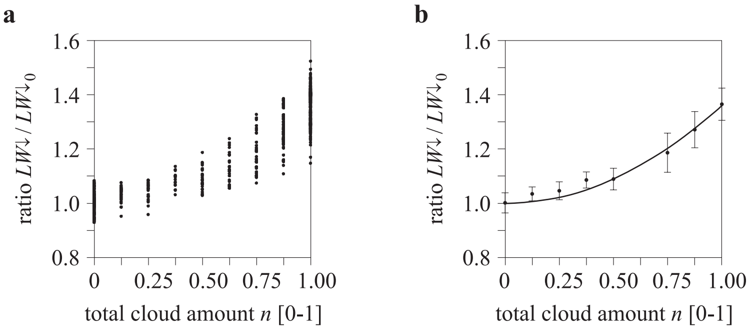

Figure 5. Empirical dependence on the total cloud amount n of the ratio of the downward longwave radiation flux for all sky conditions $LW\downarrow$ to the downward flux from a cloudless atmosphere $LW\downarrow_0$ calculated from eq. (4) (a); the same for averaged values and standard deviations of the total cloud amount in the various intervals of n (b). The solid line was calculated from eq. (5)

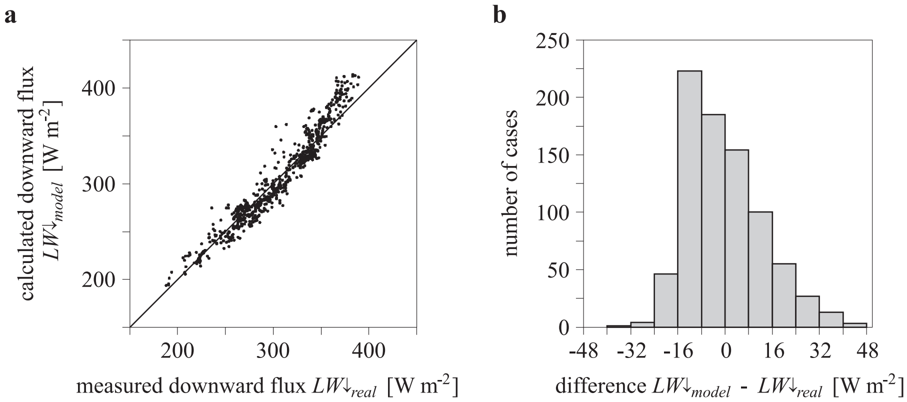

Figure 6. Comparison of measured values of the downward longwave radiation flux $LW\downarrow_{real}$ with values $LW\downarrow_{model}$ calculated using eq. (5) for all sky conditions (a); error histogram (b)

where $d = 0.36$ is a dimensionless empirical coefficient. Figure 6 compares measured values of the downward radiation flux $LW\downarrow_{real}$ with its values $LW\downarrow_{model}$ calculated using eq. (5). MBE and RMSE were estimated at 0.02 W m^{-2} and 13 W m^{-2} respectively, and the correlation coefficient $r = 0.96$. The results of this empirical verification are close to those obtained by Bignami et al. (1995).

By classifying clouds on the basis of their altitude in the troposphere (low-, mid- and high-level; see the International Cloud Atlas), it was possible to obtain a smaller scatter of the dependence of $LW\downarrow$ on the cloud cover. Figure 7 shows the same relationship as in Figure 5a, but with the clouds

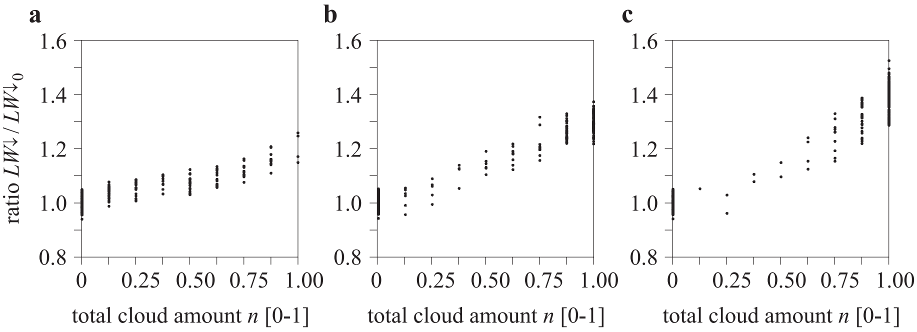

Figure 7. Empirical dependence on the total cloud amount n of the ratio of the downward longwave radiation flux for all sky conditions $LW\downarrow$ to the downward flux from a cloudless atmosphere $LW\downarrow_0$ calculated from eq. (4): low-level clouds (a), mid-level clouds (b), high- level clouds (c)

classified into these three types. It shows that despite the division into low-, mid- and high-level clouds, there is still quite a wide scatter of points within one level. This scatter is the result of a simplification: ascribing to a given group of clouds a conventional range of heights at which they can occur (International Cloud Atlas). In such a group there may be cloud types with different base heights and emissive properties. Depending on latitude, the

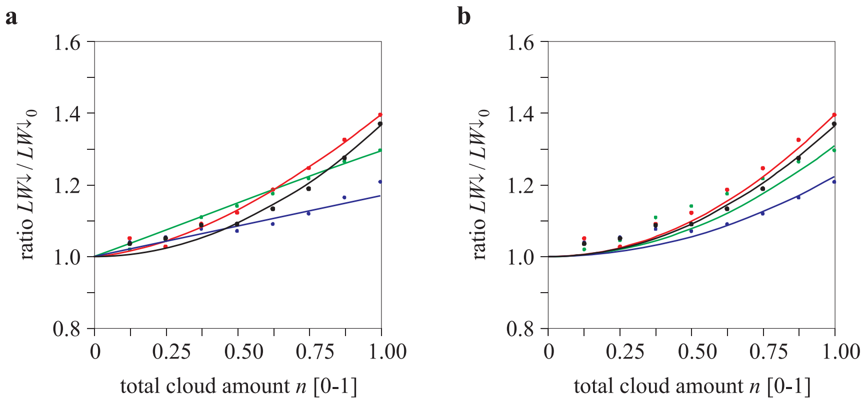

Figure 8. Mean values for particular cloud cover states of the ratio of the downward longwave radiation flux for all sky conditions $LW\downarrow$ to the downward longwave radiation flux from a cloudless atmosphere $LW\downarrow_0$ calculated from eq. (4) in relation to the total cloud amount n. The plots represent functions with different coefficients γ_i (see eq. (6) and Table 3), which best approximate the above relationship for: low-level clouds – red line, mid-level clouds – green line, high-level clouds – blue line (a); the same for the quadratic function (b). The solid black line is calculated from eq. (5)

heights of the various levels may differ, but the types of clouds within one level remain the same.

The following function was tested for each type:

$$LW\downarrow_i = \sigma T_a^4 f(e)(1 + d_i n^{\gamma_i}), \tag{6}$$

where d_i is the sought-after coefficient for clouds: $i = 1$ – low-, $i = 2$ – mid-, $i = 3$ – high-level clouds. The coefficients d_i and γ_i were determined by non-linear regression. As it turned out, both coefficients d_i and coefficients γ_i depend on the cloud base height (see Figure 8).

Table 3 lists the values that we determined of coefficients d_i and γ_i, coefficient d, and the approximation errors. It also shows the coefficients d_i for the quadratic function. These coefficients cannot be treated as mean values for the several cloud levels since, as we mentioned earlier, the cloud base heights used in the analysis may differ within one level. In addition, the values of d are undoubtedly associated with the empirical material used to determine them.

Table 3. Empirical coefficients of cloud cover functions for the Baltic appearing in formulas (6) and (7), and the absolute RMSE and MBE of the downward longwave radiation flux calculated using these formulas

Clouds	Coefficient			Error (with γ, d)		Error (with d, $\gamma = 2$)	
	γ	d	$d(\gamma = 2)$	MBE	RMSE	MBE	RMSE
high-level	0.96	0.17	0.22	–0.6	7.7	–5	8.6
mid-level	1.1	0.29	0.305	0.1	8.9	–1.9	11.3
low-level	1.6	0.39	0.39	0.6	10.1	0.1	10.8
all types	2	0.36	0.36			0.02	13.1
Total				**0.1**	**9.2**	**–1**	**9.8**

Figure 9 compares measured values of the downward components of the net longwave radiation flux $LW\downarrow_{real}$ with values of $LW\downarrow_{model}$ calculated by eq. (6) using the best approximations for each cloud type. The classification of clouds into low-, mid- and high-level clouds has considerably improved the reliability of the results: RMSE was reduced from 13 W m^{-2} to c. 10 W m^{-2}. This analysis amply demonstrates the connection between the downward longwave radiation flux, and the cloud cover and cloud type. It also shows that the cloud cover function is strictly related to cloud type.

Values of coefficients d_i determined for the quadratic function decrease with increasing cloud altitude (Table 3); the same applies to functions with different coefficients γ.

Obviously, the choice of the formula to be used to determine $LW\downarrow$ depends on the quality of the cloud cover data. If data on both cloud

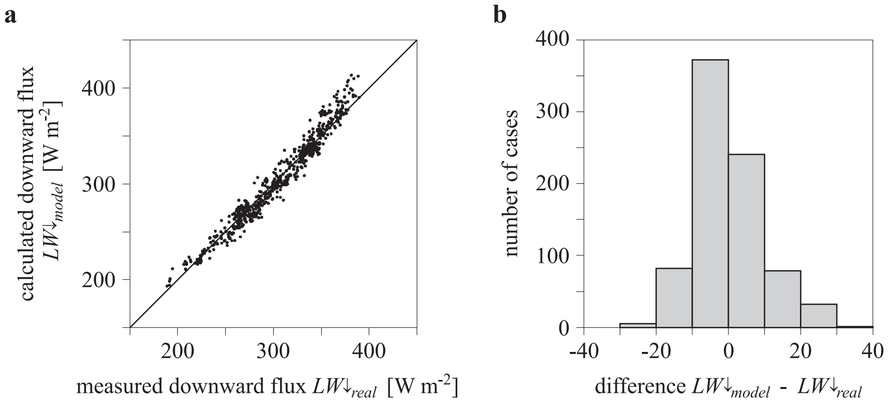

Figure 9. Comparison of measured values of the downward longwave radiation flux $LW\downarrow_{real}$ with values $LW\downarrow_{model}$ calculated using formula (6) with different γ_i (see column 2 in Table 3) (a), and error histogram (b), for all sky conditions

cover and cloud base height are available, then formula (6) can be used (see Table 3). If, on the other hand, only cloud cover data are accessible, then the quadratic function (5) with the appropriate coefficient d should be applied. On the basis of empirical data for the Baltic Sea, this coefficient has been fixed at $d = 0.36$. The reader should bear in mind, however, that the cloud cover data obtained on board ship and used here do not represent the whole Baltic Sea or all the seasons of the year with exactly the same weight. Nevertheless, a value of d representative of the entire Baltic can be obtained by analysing the cloud cover over the whole Baltic Sea. On the basis of such data as are available for the whole Baltic region and the coefficients d_i that we determined for low-, mid- and high-level clouds, we were able to estimate mean values of d for each month of the year. To this end we used Karlsson's (2001) cloud cover data for the Baltic Sea: he developed the SCANDIA model, which uses AVHRR data from NOAA satellites to identify particular cloud types. With this model he defined the frequency of each cloud type between 1991 and 2000 over the whole Baltic Sea basin. He presented his results on maps showing the frequency of each cloud type in each month of the year. He assigned each cloud type to a group linked with the means of detecting that type. His method anticipated situations in which the same cloud types occur in one group; for example, the tops of Cumulonimbus and Nimbostratus clouds reach the level of high Cirrus clouds and are accounted for in two groups – opaque Cirrus clouds, and precipitating clouds; the semi-transparent Cirrus group allows for the fact that lower-level clouds may be present beneath high-level clouds, even though there are separate groups for low- and mid-level clouds. In our analysis of Karlsson's (2001) maps, we removed all these different cloud types from his groups, assigned them to three new ones (low-, mid- and high-level clouds) and established their monthly frequency of occurrence. Groups within which no unequivocal division into low-, mid- and high-level clouds was possible were assumed to occur with equal frequency. Cumulonimbus and Nimbostratus clouds were put in the low-level group, even though the international classification places them in the mid-level group. Nimbostratus clouds occur during rainfall. It was observed that the quantity of longwave radiation reaching the sea surface during falls of rain or snow is similar to that when low-level clouds occur.

Our analysis of Karlsson's (2001) 10-year monthly means yielded the relative frequency of occurrence over the Baltic of low- p_1 , mid- p_2 and high-level p_3 clouds in each month of the year (see Table 4). The coefficient d was determined for each month in accordance with the relationship:

$$d_j = p_1 d_{1j} + p_2 d_{2j} + p_3 d_{3j}, \tag{7}$$

where p denotes the percentage in the cloud cover at a $0-1$ scale of low-level clouds (p_1), mid-level clouds (p_2), high-level clouds (p_3); d_j is the coefficient determined for the clouds of each altitude, i.e. d_{1j} – low-level clouds, d_{2j} – mid-level clouds, d_{3j} – high-level clouds; the index j refers to the month in question, from $j = 1$ for January to $j = 12$ for December. Table 4 lists the mean monthly values of d_j for each month in the year. The annual mean of this coefficient was estimated at 0.315; this differs from the value estimated from shipboard observations, 0.36. If, therefore, only cloud cover data are available for seasonal investigations of the Baltic Sea, we would suggest applying the coefficients d_j for the various months from Table 4 in

Table 4. Mean values of the coefficient d_j for each month of the year, calculated from the satellite data of Karlsson (2001) for the Baltic Sea region (standard deviations given in parentheses). Relative frequency cloud cover: low-level – p_1, mid-level – p_2, high-level – p_3

	January	February	March	April	May	June	July	August	September	October	November	December
d_j	0.313 (0.007)	0.314 (0.007)	0.316 (0.007)	0.318 (0.007)	0.317 (0.007)	0.313 (0.006)	0.312 (0.006)	0.309 (0.006)	0.313 (0.005)	0.323 (0.004)	0.319 (0.007)	0.318 (0.009)
p_1	0.43	0.42	0.42	0.45	0.45	0.43	0.45	0.42	0.44	0.51	0.46	0.47
p_2	0.29	0.32	0.34	0.33	0.34	0.33	0.30	0.29	0.30	0.26	0.30	0.28
p_3	0.25	0.23	0.21	0.18	0.17	0.20	0.21	0.25	0.23	0.20	0.21	0.23

the formula for calculating the flux $LW\downarrow$. The values of d_j obtained may be lower because satellite observations of clouds cannot guarantee that there are no clouds beneath the high-level clouds.

3.3. The net longwave radiation flux $LW\uparrow\downarrow$

Using the expressions for the upward longwave radiation flux $LW\uparrow$ (eq. (2)) and for the downward longwave radiation flux $LW\downarrow$ (eqs. (5) and (6)), the formula for the net flux $LW\uparrow\downarrow$ can be written down in the form of three alternative equations, differing in their cloud cover functions:

$$LW\uparrow\downarrow = 0.985\sigma T_s^4 - \sigma T_a^4(0.685 + 0.00452e)\begin{cases}(1+dn^2) & \text{for all cloud types} \quad \text{(Z1)}\\ (1+d_i n^2) & \text{for low-, mid- and high-level clouds} \quad \text{(Z2)}\\ (1+d_i n^{\gamma_i}) & \text{for low-, mid- and high-level clouds} \quad \text{(Z3)}\end{cases} \tag{8}$$

where σ – Stefan-Boltzmann constant; T_s – sea surface temperature [K]; T_a – air temperature [K]; e – water vapour pressure [mbar]; n – total cloud amount $[0 - 1]$; d – mean empirical dimensionless coefficient, determined for all cloud types or for particular months (Tables 3 and 4); d_i – empirical coefficient determined for the quadratic function: $d_1 = 0.39$ for low-level clouds, $d_2 = 0.305$ for mid-level clouds, $d_3 = 0.22$ for high-level clouds; d_i – empirical coefficient $d_1 = 0.39$ determined for low-level clouds when $\gamma_1 = 1.3$, $d_2 = 0.29$ determined for mid-level clouds when $\gamma_2 = 1.1$; $d_3 = 0.17$ determined for high-level clouds when $\gamma_3 = 0.96$.

4. Discussion

The statistical analysis of the earlier formulas C74, B95, JO3 and the new ones Z1, Z2, Z3 (eq. (8)) was carried out on the basis of the data used to derive the new formula. The results are compared and set out in the tables. Table 5 shows that formula (8) fulfils the principal objective of this work, since the net longwave radiation fluxes over the Baltic calculated using are encumbered with the least error. In the case of Z1, when the presence of clouds, as with C74, B95 and J03a, is characterised solely by the total cloud amount parameter, RMSE = 12.4 W m^{-2}. The errors obtained by Bignami et al. (1995) when they applied their formula to data from the Mediterranean Sea were of a similar magnitude. Even better results are obtained with eqs. (Z2) in formula (8) (RMSE = 10.3 W m^{-2}) and (Z3) (RMSE = 9.6 W m^{-2}), precisely because they incorporate data on low-, mid-

Table 5. Verification of the formulas for calculating the net longwave radiation flux $LW\uparrow\downarrow$ for the Baltic Sea region

Symbol	MBE [W m^{-2}]	RMSE [W m^{-2}]	r^2	Reference
C74	5	16	0.86	Clark et al. (1974)
B95	15.8	26	0.83	Bignami et al. (1995)
J03a	2	14.7	0.92	Josey et al. (2003)
Z1	–0.9	12.4	0.92	the present paper
Z2	0.4	10.3	0.94	the present paper
Z3	–0.9	9.6	0.94	the present paper

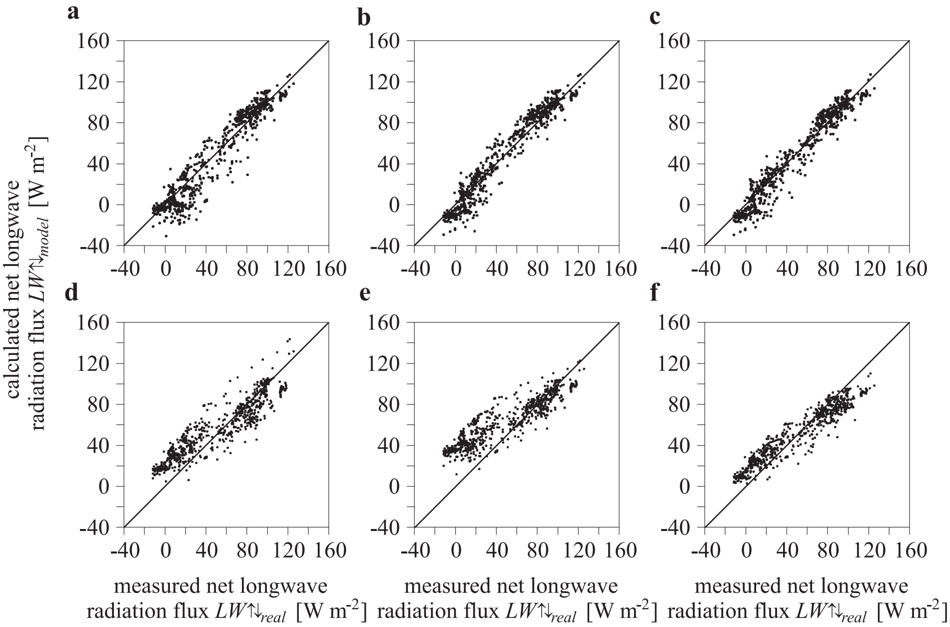

Figure 10. Comparison of values of the net longwave radiation flux $LW\uparrow\downarrow_{real}$ measured in the Baltic with values of the flux $LW\uparrow\downarrow_{model}$ calculated using the various formulas: formula (8) – eq. (Z1) (a); formula (8) – eq. (Z2) (b); formula (8) – eq. (Z3) (c); Clark et al. (1974) (d); Bignami et al. (1995) (e); Josey et al. 2003 – J03a (f)

and high-level clouds. Figure 10 compares measured values of $LW\uparrow\downarrow_{model}$ with $LW\uparrow\downarrow_{real}$ calculated according to the various formulas.

Testing these formulas further for low-, mid- and high-level clouds separately produces completely different results, however (see Table 6). It turns out that formula B95 (Table 1; Bignami et al. 1995) works very well for high-level clouds and a cloudless sky. The respective RMSE in this case

= 7 W m^{-2} and 9 W m^{-2}, and MBE = 0.7 W m^{-2} and −5 W m^{-2}. But it is quite useless for areas with prevailing low- and mid-level clouds: RMSE then increases to as much as 37 W m^{-2} in the case of low clouds. With formula J03a (Table 1; Josey et al. 2003), RMSE ranges from 9 W m^{-2} for a cloudless sky and high clouds to 7 W m^{-2} for low clouds, whereas MBE varies from −12 W m^{-2} for a cloudless sky to 15 W m^{-2} for low clouds. Formula C74 (Table 1; Clark et al. 1974) works best for mid-level clouds.

Table 6. Errors in the calculations of the flux $LW\uparrow\downarrow$ using selected formulas as applied to the Baltic Sea, depending on cloud base height

Author	MBE [W m^{-2}]			RMSE [W m^{-2}]			Correlation coefficient r^2		
	Cloud base altitude								
	high	middle	low	high	middle	low	high	middle	low
Clark et al. (1974)	–6.6	9.6	20.3	9.5	8.7	7.9	0.86	0.87	0.86
Bignami et al. (1995)	0.8	20.6	37.1	6.8	11.1	9.8	0.87	0.79	0.76
Josey et al. (2003) – J03a	–9.0	7.7	15.1	9.2	8.7	7.0	0.76	0.87	0.88
formula (8)–(Z1)	–4.6	–9.2	2.4	16.7	14.1	9.5	0.74	0.85	0.84
formula (8)–(Z2)	5.7	2.3	–4.3	8.0	11.3	10.3	0.84	0.84	0.84

Table 7. Error analysis according to water vapour pressure intervals in the air: a[0–5 mbar], b[5–10 mbar], c[10–15 mbar], d[> 15 mbar]

Author	MBE [W m^{-2}]				RMSE [W m^{-2}]				Correlation coefficient r^2			
	a	b	c	d	a	b	c	d	a	b	c	d
Clark et al. (1974)	17.4	12.0	–1.7	–6.5	12.9	12.1	15.8	12.2	0.81	0.90	0.8	0.7
Bignami et al. (1995)	25.5	25.1	7.5	2.2	21.6	18.9	18.2	11.4	0.74	0.86	0.81	0.79
Josey et al. (2003) – J03a	2.7	6.0	–2.2	1.6	17.2	13.9	15.1	8.7	0.83	0.88	0.85	0.83
formula (8)–(Z1)	6.5	3.5	–6.0	–7.2	9.5	8.1	11.9	16.9	0.88	0.92	0.86	0.86
formula (8)–(Z2)	2.6	11.8	0.7	8.5	9.0	8.1	9.9	8.3	0.88	0.92	0.86	0.85

The results of one further analysis, concerning the water vapour pressure intervals in the air (every 5 mbar) are shown in Table 7. When the water vapour pressure intervals are small (0 – 5 mbar – winter, early spring, late autumn), the errors in formulas B95 and J03a are the greatest.

Hence, these formulas yield the best results in summer, when water vapour pressures are high (15–20 mbar); then, RMSE = 11.3 W m^{-2} for B95 and 8.7 W m^{-2} for J03a. With formula (Z1) the reverse is the case (Table 7): with RMSE = 17 W m^{-2}, it provides the poorest reflection of reality in the summer; nevertheless, if coefficient d_1 defined for high-level clouds for the summer period is used in formula (Z1), this error can be reduced to 8 W m^{-2}. As with formulas B95 and J03a, C74 and B76 yield better results for water vapour pressures > 11 mbar.

5. Summary

A new, more accurate formula (8) has been derived for calculating the net longwave radiation flux $LW\uparrow\downarrow$ for the Baltic Sea. This formula is superior to other such formulas reported in the literature: it is more accurate because it incorporates a cloud cover function that discriminates the altitudes of low-, mid- and high-level clouds. Empirical verification of this formula has shown that the procedure adopted improves the accuracy of $LW\uparrow\downarrow$ by several W m^{-2}. Values of $LW\uparrow\downarrow$ determined with eq. (Z1) in formula (8), which makes use of only a total cloud amount coefficient, have RMSE = 12.4 W m^{-2}; with eq. (Z2), however, this error is reduced to 10.3 W m^{-2}. When applied to the Baltic, the other formulas stated in Table 1 produce much larger errors; in no way, however, does this detract from their utility in the regions for which they were developed. The empirical coefficients of these formulas probably typify not only the regions for which they were derived, but also the ranges of variation of the empirical data used for their derivation.

The formula expressed by eq. (Z2) may be particularly useful in satellite algorithms. The use of information on the spatio-temporal distribution of particular cloud types over the Baltic, coupled with the cloud cover functions determined in this work for these cloud types, enable the relevant coefficients characteristic of a given area or season to be defined. Coefficients of this kind were defined in this work (see Table 4). Introducing these coefficients in eq. (Z1) of formula (8) allows values of $LW\uparrow\downarrow$ to be determined on the basis of the total cloud amount for the whole Baltic with much greater accuracy than was hitherto the case. This is all the more useful in view of the fact that climatic models or satellite measurements mostly provide spatio-temporal information on the total cloud amount of the sky.

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