A review of methods for determining absolute velocities of water flow from hydrographic data*

OCEANOLOGIA, No. 31
pp. 73–85, 1991.
PL ISSN 0078-3234

Geostrophic currents
Dynamic method
Inverse methods

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Manuscript received May 9, 1991, in final form September 18, 1991.

Abstract

Methods for estimating the geostrophic velocity field from hydrographic station data are reviewed in this paper. The dynamic concepts of most of them are investigated, beginning with the so-called 'dynamic method' and ending with a three-dimensional variational inverse scheme. A short comparison of the most useful procedures in the context of their applications is given at the end.

1. Introduction

One of the classic problems of physical oceanography is to find a suitable procedure for deriving large-scale time-mean currents from observations of temperature, salinity and pressure. Unlike velocity, these quantities are relatively easy to measure, and climatological T-S fields are less contaminated by smaller-scale motions induced by eddies and waves. The aim of this paper is to review the recently developed methods for estimating the mean flow from hydrographic data.

2. Dynamics of climatological flows

The dynamics and thermodynamics describing large-scale motion in the ocean follow the classic concepts of geostrophic and hydrostatic momentum balance, vortex stretching, mass conservation and the advective balance of a conservative tracer. Noise terms can be included to allow for the effects

* The investigations were carried out as part of the research programme CPBP 03.03.C, co-ordinated by the Institute of Oceanology of the Polish Academy of Sciences.

of (i) time dependence, (ii) nonlinear interactions and eddy terms, and (iii) errors caused by the non-climatological evaluation of the density field. We are thus concerned with the set of equations, which, written in accordance with the Boussinesq approximation on the β-plane, takes the form

- momentum

$$\rho_0 f v = p_x + N_v, \tag{1}$$

$$-\rho_0 f u = p_y + N_u, \tag{2}$$

$$\rho g = -p_z, \tag{3}$$

- vorticity

$$\beta v - f w_z = N_q, \tag{4}$$

- mass

$$u_x + v_y + w_z = 0, \tag{5}$$

- tracer (heat/salt)

$$u\phi_x + v\phi_y + w\phi_z = N_\phi, \tag{6}$$

where p is the hydrostatic pressure associated with the density distribution, ρ, ρ_0 is a constant reference density, u, v, w are the eastward, poleward and upward velocities, f is the Coriolis parameter and β its northward gradient, g is the gravitational acceleration, ϕ represents potential temperature θ or salinity S or a function of these quantities such as the potential density ρ_θ; the subscripts x, y, z denote partial differentiation. The terms N_v, N_u, N_q, N_ϕ refer to noise in the assumed dynamics, and are assumed to be small.

In several cases, the vorticity balance (4), and especially the heat/salt equation (6), should be slightly modified by the inclusion of non-zero turbulent flux terms. On the other hand, equation (6), with its explicit diffusive terms, is valid for conservative tracers other than θ or S, like tritium, or a combination of nitrates or phosphates and oxygen, etc. If the constituent in equation (6) is nonconservative, then explicit source terms must be added.

Geostrophic (1), (2) and hydrostatic (3) equations can be combined into the 'thermal wind' relationship

$$u_z = g(\rho_0 f)^{-1}\rho_y, \tag{7}$$

$$v_z = -g(\rho_0 f)^{-1}\rho_x, \tag{8}$$

and vertically integrated to yield the horizontal velocity components

$$u(z) = g(\rho_0 f)^{-1}\int_{z_0}^{z} \rho_y dz' + C = u_R + u_0, \tag{9}$$

$$v(z) = -g(\rho_0 f)^{-1}\int_{z_0}^{z} \rho_x dz' + C_1 = v_R + v_0, \tag{10}$$

where

z_0 – an arbitrary 'reference level',

C, C_1 – integration constants,

u_R, v_R – relative velocities.

Clearly, the density field leaves the absolute geostrophic velocity undetermined by the constants of vertical integration (u_0, v_0), called 'reference velocites', which are subject to the geostrophic constraints

$$u_0 = -(\rho_0 f)^{-1} p_y^0, \qquad v_0 = (\rho_0 f)^{-1} p_x^0, \tag{11}$$

where

p^0 – hydrostatic pressure at the z_0 level.

The most useful procedures for determining the reference velocities (u_0, v_0), and thus the absolute geostrophic field (u, v), will be presented in the following sections. The vertical velocity component w can be derived from the vorticity (4) or tracer equation (6).

3. Methods for deducing the velocity field from hydrographic data

3.1. Dynamic method

The 'thermal wind' relationships (7), (8), and the assumption of the 'level of no motion' (Mamayev, 1962) form the basis of the old 'dynamic method' for calculating the velocity of ocean currents from density field observations. At the 'level of no motion' z_0 the absolute velocities are equal to the relative velocities

$$v_0 = 0, \qquad u_0 = 0, \qquad v = v_R, \qquad u = u_R. \tag{12}$$

In the real ocean the reference level for u and v may differ.

It is more convenient to express u and v in terms of the 'dynamic height' (Zubov and Mamayev, 1956)

$$D = \int_{p^0}^{p} \alpha dp', \tag{13}$$

or the dynamic height anomaly

$$\Delta D = \int_{p^0}^{p} \delta dp', \tag{14}$$

where $\alpha (= \rho^{-1})$ is the specific volume, and $\delta (= \alpha - \alpha(35, 0, p))$ its anomaly.

In this case, velocities are evaluated from the equation

$$(u, v) = (fL)^{-1}(\Delta D_B - \Delta D_A), \tag{15}$$

where $\Delta D_B - \Delta D_A$ is the difference in the dynamic height anomaly as a function of pressure between sites B and A, separated by a distance L.

Most of the attempts to determine the 'level of no motion' have been based on intuition gained from water mass analysis or on another hydrographically related criterion such as the minimum roughness of isopicnal surfaces, minimum velocity shear or oxygen minimum (Mamayev, 1962).

By constructing a circulation pattern for a region enclosed by a track of hydrographic stations, one attempts to satisfy the requirement of the mass conservation of M apparently independent layers. The velocity field obtained by the 'dynamic method' usually fails in this respect. Let v_r be the 'dynamic method' velocity perpendicular to the cruise track at the midpoint of $\Delta_i x$ of the i-th station pair ($i = 1, 2, ..., N$) and T_{rj} the total volume flux for the j-th layer. Then

$$T_{rj} = \sum_{i=1}^{N} \int_{z_{j-1}}^{z_j} v_{ri} dz \Delta_i x, \qquad j = 1, 2, ..., M. \tag{16}$$

To find the optimum reference level, one chooses z_0 at different depths, calculates the net transport of each layer for each choice of z_0, and forms the mean square imbalance

$$T_r^2 = \sum_{j=1}^{M} T_{rj}^2. \tag{17}$$

The value of z_0 for which T_r^2 is minimum can be considered the best choice of the 'level of no motion' obtained by this procedure (Fiadeiro and Veronis, 1982).

3.2. Inverse methods

3.2.1. The box method

The difficulties inherent in the dynamic method in meeting the requirement of mass conservation of independent layers in an enclosed region, where ranges of layers (boxes) are selected by inspection of the local density field, led to the assumption (Wunsch, 1978, 1984) that the 'level of no motion' varies from one location to another. After adding the correction $b(x)$ to the relative velocity v_r in such a way that the imbalance of each layer (T_{rj}) vanishes, equation (16) takes the form

$$\sum_{i=1}^{N} \int_{z_{j-1}}^{z_j} (v_{ri} + b_i) dz \Delta_i x = 0, \qquad j = 1, 2, ..., M. \tag{18}$$

Using (16) once again, this can be rewritten as

$$\sum_{i=1}^{N} b_i h_{ij} \Delta_i x = \sum_{i=1}^{N} A_{ij} b_i = -T_{rj}, \qquad j = 1, 2, ..., M, \tag{19}$$

where h_{ij} is the average value of the thickness of the j-th layer at the midpoint between each station pair and $A_{ij}(= h_{ij}\Delta_i x)$ is the area of the j-th layer in the i-th interval.

Equation (19) can be written as a linear underdetermined system (M is usually less than N)

$$\sum_{i=1}^{N} A_{ij} b_i = c_j, \qquad c_j = -T_{rj}, \qquad j = 1, 2, ..., M, \tag{20}$$

or in matrix notation

$$A^{(MXN)} b^{(N)} = c^{(M)}. \tag{21}$$

M could be increased by including additional tracers (6) to be conserved in the box, but it still remains less than N. The system (21) is solved by inverse analysis (Wunsch, 1978), and an optimum permissible solution for the b vector is chosen. It is usual to pick the minimum kinetic energy solution.

Wunsch (1978) and Stommel and Veronis (1981) showed that the solution of system (21) is determined by both the thickness of the layers involved and the variable station distance. To avoid such an inconvenience, Wunsch (1978) proposed filtering (21) by normalizing the columns of matrix A by the square root of their length.

To obtain more realistic inversions one can allow for vertical mass exchange (Bigg, 1985).

A more complicated formulation was derived by Mercier (1989) who combined hydrographic and current meter data and used a nonlinear inverse model which explicitly accounted for the errors in the data and the dynamics for studying the steady circulation in the western North Atlantic.

3.2.2. The beta-spiral method

The original beta-spiral method is fully discussed in Schott and Stommel (1978) and in Stommel and Schott (1977) and assumes the validity of 'thermal wind' balances (7), (8) and approximate tracer conservation (6).

Following Olbers *et al.* (1985), one can differentiate the tracer balance (6) with respect to z, then combine it with the 'thermal wind' relationships (7), (8) and the vortex stretching equation (4) and get the potential vorticity equation

$$uQ_x + vQ_y + wQ_z = g\rho_0^{-1} J_{xy}(\rho, \phi), \tag{22}$$

where $Q(= f\phi_z)$ is the potential vorticity and $J_{xy}(\rho, \phi)$ $(= \rho_x\phi_y - \rho_y\phi_x)$ is the z-th component of the Jacobian operator.

Usually, the potential density serves as the known seawater property $(\phi = \rho_\theta)$, but one can use other constituent distributions like those of chemical tracers (Bigg and Killworth, 1988).

Equation (22) can serve as an additional tracer balance in the box method.

Elimination of w between (6) and (22) yields

$$uJ_{xz}(Q,\phi) + vJ_{yz}(Q,\phi) = g\rho_0^{-1}J_{xy}(\rho,\phi)\phi_z. \tag{23}$$

Now, it is possible to choose the reference level for the integrated thermal wind relationships (9) and (10), insert them into (23), and obtain an equation for reference velocities (u_0, v_0)

$$a_1u_0 + b_1v_0 + c_1 = 0, \tag{24}$$

where a_1, b_1, c_1 are coefficients depending only on measured values.

Considering equation (24) with two unknowns at two different levels ($N = 2, M = 2$) yields a unique solution. In practice, because of noisy data, a more stable solution is obtained by writing (23) at more than two levels and determining (u_0, v_0) from the resulting overdetermined set of equations by the least-squares principle

$$a_ju_0 + b_jv_0 + c_j = R_j, \qquad 1 <= j <= M, \qquad M > 2, \tag{25a}$$

$$\sum_{j=1}^{M} R_j^2 = \min, \tag{25b}$$

where

R_j – the imbalance of the j-th equation.

It can be shown (Davis, 1978) that problems (21) and (25) are based on the same dynamic sophistication. The differences between them result from implicit assumptions about the scales of oceanic variability and different definitions of the smooth field for which the dynamic model is suitable. The selection of an unknown at each station in the box method is equivalent to assuming the scale of the smooth field to be comparable with the station spacing. In the beta-spiral method, the smooth field has scales larger than the station spacing. Their magnitude depends on the distances between stations taken to calculate the density gradients at the station for which system (25) is constructed.

From the practical point of view it is more desirable to construct a modified beta-spiral scheme from the same set of equations that leads to (25) (Bigg, 1985; Olbers *et al.*, 1985). Such a scheme involves derivatives only up to the first order and three unknowns (u_0, v_0, w_0), where w_0 is the vertical velocity at $z = z_0$. This procedure requires the elimination of w between the tracer balance (6) and the integrated vorticity equation (4) and the substitution of u and v in the resulting equation with the integrated 'thermal wind' relationships (9) and (10). The equivalent of (24) then becomes

$$au_0 + bv_0 + cw_0 + d = 0, \tag{26}$$

where

$$a = \phi_x, \qquad b = \phi_y + \beta f^{-1}(z - z_0)\phi_z, \qquad c = \phi_z, \tag{27}$$

$$d = u_R\phi_x + v_R\phi_y - (\beta g f^{-2}\rho_0^{-1}\int_{z_0}^{z}(z - z_0)\rho_x dz')\phi_z, \tag{28}$$

and the equivalent of (25) remains overdetermined for $M > 3$.

The effect of horizontal and vertical diffusion can also be included in (6). In such a case, one can derive a system similar to (25), but with more unknowns (Bigg, 1985; Olbers *et al.*, 1985).

It is worth stressing here that the beta-spiral method is a local calculation because it determines the velocities for each horizontal location (x, y) separately. Killworth and Bigg (1988) required the conservation of mass between stations, and thus linked together separate overdetermined systems.

It seems that the beta-spiral method can be adopted for synoptic fields (Korotayev and Shapiro, 1978). Such a procedure demands the inclusion of time-dependent terms in the vorticity (4) and tracer (6) balance and relative vorticity terms in (4) which would increase the number of unknowns in system (25).

3.2.3. The Bernoulli method

The Bernoulli method (Killworth, 1986) assumes that a steady-state ocean is in approximate geostrophic (1), (2) and hydrostatic balance (3), that it conserves mass (5) exactly and density (6) approximately. In such a case, three scalars – the density ρ, the linear potential vorticity $Q(= f\rho_z)$ and the Bernoulli function

$$B = p + \rho g z, \tag{29a}$$

$$B_z = z g \rho_z, \tag{29b}$$

are approximately conserved. Involving a pair of stations, the method requires such depths to be found – (z_A) at the first station and (z_B) at the second, where ρ and Q are matched (the search begins below the depth D of the mixed layer so that Q is well defined). At these depths B also matches. Thus, after the integration of (29b), we can get

$$B = B^{-D} - g\int_{z}^{-D} z\rho_z dz, \tag{30}$$

and then

$$B_A^{-D} - g\int_{z_A}^{-D} z\rho_z dz = B_B^{-D} - g\int_{z_B}^{-D} z\rho_z dz, \tag{31}$$

where B^{-D}, B_A^{-D} and B_B^{-D} are the constants of integration.

Typically, there are typically several matches between any two stations, so that we are dealing with a large overdetermined system of equations for the set of B^{-D} as the unknowns. It is solved like system (25) in the usual least-squares manner. The pressure field is then evaluated from (30) and (29a) and the horizontal velocities calculated from the geostrophic balance (11).

3.2.4. Variational inverse methods

The determination of the velocity field can be formulated in the calculus of variations where one attempts to minimize a certain functional.

In their box (multi-layer) variational inverse method Stommel and Veronis (1981) minimized the reference velocity amplitude (b^2) integrated around the sides L of a square

$$\int_0^L b^2 dx = \min, \tag{32}$$

subject to transport constraints (19). Such an approach yields b as a linear combination of the layer thicknesses with the coefficients determined by the transport constraints.

For discrete locations around L the integral (32) becomes a sum and the solution cannot give information other than that supplied by the box system (21).

The beta-spiral problem of mass conservation between stations can be solved by allowing variations of the reference pressure $p^0 = p^0(x, y)$ over the whole domain of interest V (the volume of the fluid).

The substitution of geostrophic constraints (11) in the relationship between reference velocities (24) and the introduction of a misfit $R(x, y, z)$ (an imbalance) yield a differential equation for p^0

$$-a_1(f\rho_0)^{-1}p_y^0 + b_1(f\rho_0)^{-1}p_x^0 + c_1 = R. \tag{33}$$

Now, the sought-after function p^0 may be defined as the solution of the three-dimensional variational problem (Zhdanov and Kamenkovich, 1984, 1985)

$$J(p^0) = \int_V R^2 dV = \min, \tag{34}$$

and the reference velocities (u_0, v_0) obtained from (11).

If the data are available over the whole range of depths, the usual boundary condition of no flow through the bottom leads to

$$w_0 = w^{-H} = -u_0 H_x - v_0 H_y \qquad \text{at} \qquad z = -H, \tag{35}$$

where H is the depth of the ocean.

To write equation (26) in the form of (33), condition (35) is applied, and finally

$$a' p_y^0 + b' p_x^0 + d' = R, \tag{36}$$

where

$$a' = -a + \phi_z H_x, \qquad b' = b - \phi_z H_y, \qquad d' = df\rho_0. \tag{37}$$

The differential nature of equation (36) requires the boundary conditions to be specified. To retain a simple formulation for the method, one may assume

$$p^0 = 0 \qquad \text{on} \qquad \delta D, \tag{38}$$

where D denotes the lateral boundary.

The Euler equation associated with the problem (34), (36) and (38) was given by Peggion (1988) in the form

$$L(p^0) = 0 \qquad \text{and} \qquad p^0 = 0 \qquad \text{on} \qquad \delta D, \tag{39}$$

where

$$\begin{aligned} L(p^0) &= [< (a')^2 > p_y^0]_y + [< (b')^2 > p_x^0]_x + \\ &+ (< a'b' > p_x^0)_y + (< a'b' > p_y^0)_x + G, \end{aligned} \tag{40a}$$

$$< ... > = \int_{-H}^{0} ...dz \qquad \text{and} \qquad G = < a'd' >_y + < b'd' >_x . \tag{40b}$$

If M_f field distributions are known, equation (36) is written as

$$a_k' p_y^0 + b_k' p_x^0 + d_k' = R_k, \qquad k = 1, 2, ..., M_f. \tag{41}$$

Thus, the variational problem (34) can be redefined as

$$J^*(p^0) = \sum_{k=1}^{M_f} \Theta_k \int_V \Gamma_k R_k^2 dV = \min, \tag{42}$$

where the constants Θ_k allow to rank dynamically one constituent equation above the others, and the weight functions Γ_k are the mechanisms for correcting and adjusting each data set.

The numerical scheme to be chosen in the treatment of equation (39) again leads to the inversion of hydrographic data.

A fully three-dimensional variational method requiring solutions with minimum roughness (energy or entropy) was developed by Provost and Salomon (1986). The aim of that method is to explore the envelope of geostrophic flow fields which are consistent with hydrographic data and the imposed dynamical constraints (1–6) to within prescribed misfits (N_v, N_q, N_q, N_ϕ).

Let $\Psi(x)$ be an abstract flow variable somehow connected with properties measured at discrete locations $\{x_i, \ i = 1, 2, ..., N_d\}$ in three-dimensional space (V), and the functional R or R_1 a measure of the roughness of $\Psi(x)$

$$R(\Psi) = \int_V (\nabla^2 \Psi)^2 dV, \tag{43}$$

$$R_1(\Psi) = \int_V \nabla\Psi \nabla \Psi dV. \tag{44}$$

The measurement d_i of $\Psi(x_i)$ is aliased and the error is

$$e_i = d_i - \Psi(x_i). \tag{45}$$

If q_0 is the expected error in a single measurement of $\Psi(x)$, then

$$\sum_{i=1}^{N_d} e_i^2 = q_0^2 N_d. \tag{46}$$

If the dynamic constraints (1–6) or their combinations written in the form

$$C_j(\Psi) = 0, \qquad j = 1, 2, ..., M_e, \tag{47}$$

are approximately satisfied, one obtains

$$\int_V C_j(\Psi)^2 dV = q_j^2 V, \tag{48}$$

where

C_j – any operator,
M_e – number of equations,
q_j – the expected error (misfit) in the j-th equation, usually estimated by a scaling analysis,
V – the volume of fluid.

The method consists of minimizing the roughness R or R_1 subject to constraints (46) and (48). This is a problem in the calculus of variations, and the functional to be minimized (for R) becomes

$$J(\Psi)=\int_V \{(\nabla^2\Psi)^2 + \lambda_0 \sum_i (\Psi - d_i)^2 \delta(x - x_i) + \sum_j \lambda_j C_j(\Psi)^2\} dV, \tag{49}$$

where λ_0 and $\{\lambda_j\}$ are the Lagrange multipliers uniquely determined by the misfits q_0 and $\{q_j\}$.

If one puts $\Psi(x) = p/g$ (Provost and Salomon, 1986), then all variables in (1–6) are either known from measurements or expressible in terms of Ψ, and because of the geostrophic relations (1) and (2), R_1 becomes the kinetic energy and R the entropy. The minimum of (49) has to be found numerically, which leads to the inversion of a certain matrix depending on the numerical representation of Ψ.

The advantage of such a variational formulation is that the relative emphasis on smoothing, agreement the data and the dynamic constraints can be altered by adjusting the Lagrange multipliers.

3.3. Other methods

There are two other interesting methods for determining absolute velocities from hydrographic data: the single hydrographic section method, and that of 'minimum action'.

Killworth's method for a single north-south section (*e.g.* 1980, 1983) assumes precisely the same dynamics as does the beta-spiral method, but is more ambiguous. The problem is reduced to a linear second-order differential equation in the vertical for the vertical velocity.

Fomin (1984, 1985) treated the problem of determining the absolute velocities of geostrophic flows by the assumption of 'minimum action'. According to his hypothesis, when the motion is in geostrophic and hydrostatic balance (1–3), the velocity field has to fulfil the continuity equation and is related to density distribution, surface elevation and topography in such a way that the vertically integrated kinetic energy

$$E = 0.5\rho_0 \int_{-H}^{0} (u^2 + v^2) dz = 0.5\rho_0 \int_{-H}^{0} [(u_R + u_0)^2 + (v_R + v_0)^2] dz, \tag{50}$$

assumes its minimum for each horizontal location (x, y) (Nefedev, 1976).

The boundary conditions for the vertical component of velocity w at the sea surface level and at the bottom can take the form

$$w = 0 \quad at \quad z = 0, \tag{51}$$

$$w = -uH_x - vH_y \quad at \quad z = -H. \tag{52}$$

The vorticity equation (4) integrated from $z = -H$ to $z = 0$ taken along with (9), (10), (51) and (52) leads to the equation

$$Au_0 + Bv_0 = -\beta H f^{-2} \bar{v}_R + f^{-1}(u_R^{-H} H_x + v_R^{-H} H_y), \tag{53}$$

where

$$A = -f^{-1} H_x, \qquad B = -f^{-1} H_y + \beta H f^{-2}, \tag{54}$$

u_R^{-H}, v_R^{-H} – relative velocities at $z = -H$, and $\bar{v}_R$ – vertical mean of the longitudinal relative velocity component v_R.

Now, one can take v_0 (or u_0) from (53) in (50), and after integrating differentiate it with respect to u_0 (v_0). From the relative minimum condition of (50) ($\delta E/\delta u_0$ or $\delta E/\delta v_0 = 0$) it is then possible to get the reference velocities dependent only on density distribution and topography

$$u_0 = -(A^2 + B^2)^{-1} [B^2 \bar{u}_R + A^2 u_R^{-H} - f^{-1} H_y A (v_R^{-H} - \bar{v}_R], \tag{55}$$

$$v_0 = -(A^2 + B^2)^{-1} [-(A^2 + \beta H B f^{-2}) \bar{v}_R + f^{-1} H_y B v_R^{-H} - {} + AB(u_R^{-H} - \bar{u}_R)]. \tag{56}$$

Although simple, this method is a little ambiguous because it is based on the hypothesis of the minimum kinetic energy of a single profile.

4. Conclusions

Although for certain data sets a satisfactory estimate of the 'level of no motion' can be obtained by empirical search, it is evident that, in the general case, mass conservation requires the reference correction of the horizontal velocity components. As long as computing possibilities are promoting the inverse methods as opposed to the 'dynamic method', they are becoming more and more powerful. However, their applications may be limited. The beta-spiral method is fundamentally a local calculation determining the three components of velocity as functions of depth at a single geographical location. The method cannot be expected to hold in regions of strong currents with large relative vorticity, where the potential vorticity is no longer constant along flow lines. The Bernoulli method can handle data over a much wider range and does not require horizontal differentiation of the density field but fails in homogeneous regions where the potential vorticity is not well defined. The inverse box method aims at giving a reliable description of the oceanic flux, but requires a complete set of data over a closed volume of water. Variational formulations, like that of Provost and Salomon, seem to be the only fully three-dimensional methods. They make it possible to assess the quality of various dynamic constraints.

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