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2/7

On some statistical characteristics of the sea water temperature variations at the Polish Baltic coast *

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Abstract

In the present paper the periodic structure of the sea water temperature variations is determined, the usefulness of process AR(1) is analysed, and the distribution functions of the temperature range variations in individual summer months are computed using the Monte Carlo method. The computations were based on the series of daily water temperature measurements at the stations at Międzyzdroje, Mielno, and Władysławowo in the years 1961 – 1970, as well as on monthly mean values for these stations in the period of 1950 – 1984. The computations were carried out having in mind the optimum use of the results for a short period forecast with a 24-hour lead time, and for the characteristics of the summer season.

1. Introduction

The temperature of sea water is one of the fundamental parameters related to the physical processes taken as a whole and occurring in an aqueous environment. Despite its significance, the possibilities of systematic investigation of a sea water temperatures within long time periods are limited to the data obtained from coastal measuring stations, due to technical measuring difficulties. The measurements at such stations give the temperature characteristics of the coastal surface waters differing in principle from those of the open sea surface waters.

The sea water temperature variations in the coastal near-shore zone depends on numerous influencing factors. A small depth of water causes that the exchange of heat takes place in a thin layer. The type of the coast *ie* the inclination of the sea bottom from shoreline toward the open sea is under those conditions a very important factor. Variations of the radiation, heat

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balance of the sea surface, turbulent mixing processes which are connected with local currents, the waves, and the thermal gradient are the next factors. The influence of the open sea is also important. It is connected with advection of surface waters tied closely with general field of currents which is controlled mainly by wind field. Upwellings also belong here being caused mainly by wind field and affected by vertical stratification of the sea.

Rapid changes of water temperature are related to the inflow of water masses with different temperatures. This may concern surface waters or deep waters uplifted towards the surface under favourable hydrodynamic conditions. Such displacements of water masses are caused by strong winds. A physical properties mentioned above create the sea water temperature in near-shore zone a separate subject of studies (Hupfer, Lass, 1975; Lozovatskii 1978; Walin, 1972).

Among the papers regarding the above problems one can find the works considering a general characteristics of the Polish coast (Dziadziuszko, 1961) as well as the Gulf of Gdańsk (Cyberska, Trzosińska, 1984; Majewski, 1979). The characteristics of the sea water temperature variations can also be found in monographs elaborated for individual water basins of the Polish coast.

The computations presented in this paper were carried out on the basis of daily measurements of water temperature at 12.00 GMT in the years 1961 – 1970 (Hydrographic Annals of the Baltic Sea) and on monthly mean values from measurements in the period 1950 – 1984 (Hydrographic Annals of the Baltic Sea; Maritime Hydrological and Meteorological Report)*. The measurements were performed 0.5 m under the surface of the water at coastal stations at Międzyzdroje, Mielno, and Władysławowo. Geographical positions of the measuring stations are shown in Figure 1.

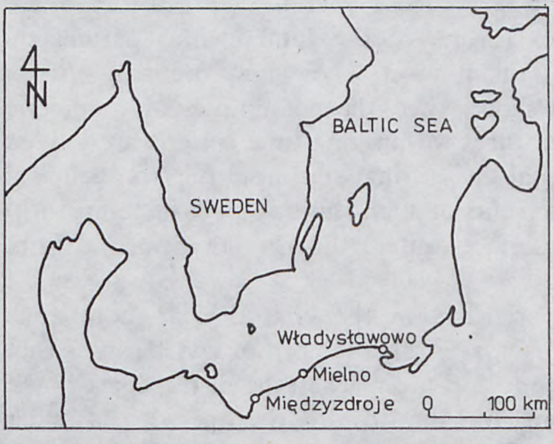

Fig. 1. Geographical position of the measuring station

* The observation at Mielno covered the period 1950 – 1978.

2. Periodic structure

In the first step the basic statistical characteristics were computed together with the autocorrelation function for the first time lags. Respective data are shown in Table 1 for a sample size of the time series $N = 3652$.

Table 1. Main statistical characteristics of daily sea water temperatures at Międzyzdroje, Mielno, and Władysławowo based on the measurements from 1961 to 1970

Station	$\bar{x}$ [°C]	σ_x [°C]	$R_x(1)$	$R_x(2)$	$R_x(3)$	x_{min} [°C]	x_{max} [°C]
Międzyzdroje	9.1	6.8	0.99	0.99	0.99	−0.5	23.0
Mielno	8.8	6.3	0.99	0.98	0.97	−0.5	22.8
Władysławowo	8.2	6.2	0.99	0.99	0.98	−0.5	20.8

Correlation coefficients between individual data series range from 0.97 to 0.98 which enables the periodic structure to be determined according to a single measurement series.

Spectral density function was computed for a series from Władysławowo, assuming maximum correlation lag $M = 182$. The computation results are given in Figure 2. The spectral density function shows no marked periodicity. The type of the spectrum presented can be computed analytically from the formula:

$$S_x(f) = \frac{\sigma_x^2}{2\pi} \int_{-\infty}^{\infty} e^{-if\tau - \alpha|\tau|} \, d\tau, \tag{1}$$

where:

$S_x(f)$ – spectral density function

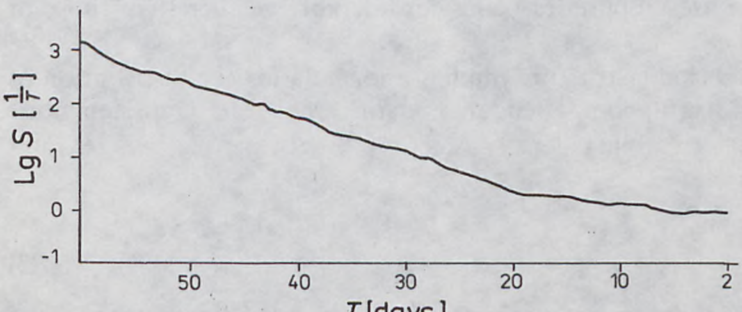

Fig. 2. Spectral density function of daily sea water temperatures at Władysławowo according to the measurements in the years 1961 – 1970

σ_x^2 – variance of a random process,
α – coefficient of the curve shape,
The autocovariance function corresponding to the function given by formula (1) is determined by:

$$V_x(\tau) = \sigma_x^2 \, e^{-\alpha|\tau|}. \tag{2}$$

The course of the plot implies only that exists a possibility of significant oscillations in the interval beyond the resolution assumed. The computations carried out were completed with those of mean monthly water temperatures in Władysławowo in the years 1950–1984. The sample size of the new data series was $N = 420$ and maximum correlation lag $M = 42$ was assumed. The results given in Figure 3 show very strong one-year period. In the frequency intervals, which are not included in the above period, the computations yield only noises of negligible amplitudes.

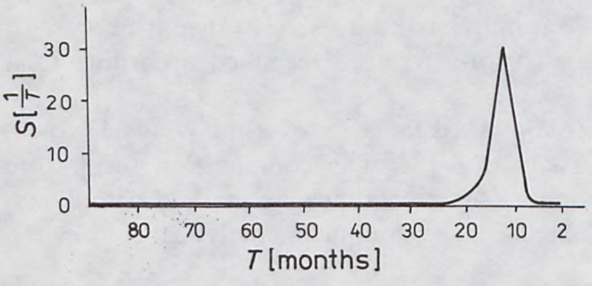

Fig. 3. Spectral density function of mean monthly sea water temperatures at Władysławowo according to measurements in the years 1950–1984

The computations carried out when taking into acount the course of the autocorrelation functions and spectral density functions imply unequivocal and simple image of the periodic structure of the sea water temperature variations at the Polish coast. These changes can be interpreted as those determined by a sine wave with one-year period, with the Gaussian noise superimposed, the noise amplitudes being considerably smaller than those of the sine wave.

In this case the periodic structure implies a formula for the distribution of the probability density function (Bendat, Piersol, 1983). The Gaussian noise has the well known probability density function. Sine wave has a deterministic formula:

$$x(t) = X \sin(2\pi f t + \theta). \tag{3}$$

If the phase angle θ is a random variable with a uniform distribution in the interval $\pm\pi$, then the sine wave can be treated as a random process. The probability density function of this process has a formulae:

$$p(x) = (\pi \sqrt{2\sigma_s^2 - x^2})^{-1}; \; |x| < X,$$
$$p(x) = 0; \; |x| \geqslant X, \tag{3a}$$

where $\sigma_s = X\sqrt{2}$ is the standard deviation of the sine wave.
The above mentioned function has the 'U-shaped' characteristic. If the process is a sine wave with an addition of Gaussian noise, then the probability density function is a convolution of the formula (3a) with the equivalent Gaussian distribution formula. Then the shape of the curve depends on the ratio of a sine component to noise. A predominant noise component creates the curve shape similar to the Gaussian bell when as strong an influence of the sine wave is modelling the 'U-shaped' curve. Intermediate shapes are also possible in practice.

According to above mentioned formulae and former computations, the distribution of the probability density function of the phenomenon analysed is determined by the following equation:

$$p(x) = \frac{1}{\sigma_n \pi \sqrt{2\pi}} \int_0^{\pi} \exp\left[-\left(\frac{x - S\cos\theta}{4\sigma_n}\right)^2\right] d\theta, \tag{3b}$$

where:
σ_n – standard deviation of the Gaussian noise,
S – amplitude of one-year period,
θ – phase of one-year period.
The computations of the empirical probability density function showed the existence of a 'U-shape' plot corresponding to high values of coefficient $R_0 = \sigma_s^2/\sigma_n^2$ characterizing the ratio of the sine to the noise components.

Practical conclusions arising from the analysis carried out show the mean value for the distribution presented to be the least probable one. The standard deviation obtained using the mean value should also be interpreted differently from that computed from the distributions where the mean value is most probable. It could be anticipated that the determination of the sine wave components would enable a highly accurate temperature forecast and that the accuracy could be improved by an addition of the noise component occurring in the probability intervals of the normal distribution. Such conclusion is incorrect since the obtained regularity in the sine component, resulting from one-year distribution of the energy influx, is due to the averaging after the realization time. As is generally known, the changes in the air and the surface water temperatures during individual years are considerably diversified, and the harmonic characteristics are variable. Besides, the effect of solar energy, that of the remaining factors, influencing less significantly the water temperature changes within long periods, may be of essential importance in certain cases of a short-period component.

The values of the autocorrelation function summarized in Table 1 cannot be applied to the autoregressive scheme since they are too close to unity. Such a strong regression is due to the small data sampling step with respect to the variability of the phenomenon as well as to the occurrence of a predominant sine wave. In the computations the long-term trend mentioned in papers concerning the Baltic water temperatures (Matthäus, 1982) was not taken into account due to a relatively short measuring period.

3. Analysis of the usefulness of process AR(1)

As already mentioned, the values of the autocorrelation function given in Table 1 make the use of the autoregression properties used recently in many publications (*eg* Barnett, Hasselman, 1979, 1984) impossible. However, a different approach to the employment of autoregression is possible. This method was considered previously, assuming a stationary ergodic random process. Let us assume that the temperature course in individual months constitutes the realization of transient random processes with sample sizes of $N_q = 28$ to $N_q = 31$, the repeatability of the process being $n = 10$, and the separations between individual realizations corresponding to 12 months. The characteristics of individual 12 processes will be averaged over n realizations. Thus, the course of the sine wave in one-year period will be replaced by the trend in the transient process or a process without seasonal trend, corresponding to months situated at the inflexions of the annual wave.

If general linear relations between the predictand and the predictors are expressed according to the formula for many predictors with many time lags and the condition that the predictors may but need not be autocorrelative, relations (5) will be obtained:

$$\hat{x}(t_j + p\Delta t) = \sum_{i=1}^{l} \sum_{k=0}^{m-1} D_{ik}\, y_i(t_j - k\Delta t), \tag{5}$$

where:

$x(t_j)$ – predictand,

Δt – time step of the prediction,

p – number of time lags for the future, depending on the present and the past values of the predictor series $y_i(t_j)$, $(i = 1, \ldots, l)$,

D_{ik} – weighting coefficients.

The above formula is a numerical solution to a known linear system of constant parameters with one output and many inputs. For seasonal relations occurring with one-year period T, the generalization of the physical system by introducing the Fourier series which replaces constant values D_{ik} in formula (5) is more appropriate:

$$D_{ik}(t_j) = \sum_{a=-\infty}^{\infty} D_{ikq} \exp(i2\pi a t_j / T), \tag{6}$$

The use of transient random processes for individual months and processes AR(1) simplified the formulae (5) and (6) to the following expression:

$$x_q(t_j) = D_q x_q(t_j - 1) + \varepsilon_q(t_j), \tag{7}$$

where:

$x_q(t_j)$ – transient random process ($q = 1, 2, \ldots, 12$),

D_q – weighting parameters of AR(1),

$\varepsilon_q(t_j)$ – stochastic component $N(0, \sigma_{\varepsilon_q})$.

The results of computations according to the model AR(1) applied to individual months are presented in Table 2.

Table 2. Application of model AR(1) for the prediction of the sea water temperature at 12.00 GMT with a 24-hour lead time for measuring stations in Międzyzdroje (1), Mielno (2), and Władysławowo (3)

q	x_q [°C]			σ_{x_q} [°C]			D_q			σ_{ε_q} [°C]		
	1	2	3	1	2	3	1	2	3	1	2	3
1	0.52	0.87	0.62	0.84	0.92	0.86	0.94	0.87	0.93	0.28	0.37	0.26
2	0.61	0.98	0.56	0.85	1.0	0.86	0.87	0.89	0.92	0.37	0.38	0.24
3	1.82	2.44	1.78	1.79	1.78	1.74	0.93	0.88	0.95	0.51	0.60	0.27
4	5.83	6.35	5.24	2.12	2.10	1.84	0.88	0.80	0.94	0.83	0.97	0.42
5	10.77	10.45	9.04	2.28	1.83	1.54	0.86	0.71	0.88	0.99	1.16	0.63
6	16.51	14.32	13.23	1.99	2.78	2.36	0.80	0.78	0.86	1.10	1.56	1.12
7	18.05	17.64	16.91	1.55	1.75	1.78	0.80	0.70	0.78	0.78	1.22	0.97
8	18.01	16.56	16.47	1.35	2.58	2.48	0.76	0.75	0.89	0.73	1.42	0.97
9	16.53	15.79	15.23	1.63	2.15	2.15	0.86	0.81	0.89	0.67	1.04	0.78
10	11.74	11.64	10.87	1.63	1.72	1.54	0.90	0.88	0.91	0.52	0.67	0.50
11	7.04	7.18	6.71	1.96	1.97	1.86	0.92	0.86	0.93	0.49	0.71	0.46
12	2.35	2.72	2.38	1.74	1.69	1.46	0.92	0.88	0.95	0.50	0.64	0.35

The data given therein enable the conclusion to be drawn that the standard deviations σ_{ε_q} amount on average to 41% of the σ_{xq} value determined from the measurements. This evidences a considerable effectiveness of the computations carried out by the use of the formula (7). The effectiveness of AR(1) can also be compared with the data σ_x in Table 1. The computations according to this model can be carried out by determining the probability of the occurrence of a temperature predicted in a certain interval. To this end, the values of $\varepsilon_q(t_j)$ should be assumed as those from the intervals of normal distribution, the probability of which is related to the standard deviations σ_{ε_q}.

The most effective results of the application of transient processes and AR(1) were obtained in Władysławowo and, secondly, in Międzyzdroje.

Relatively poorer computation results for Mielno are due to the position of the measuring station located very close to the shore. This resulted in lower autoregressivity of the measuring series.

The possibilities of practical employment of the results from Table 2 are varied depending on the magnitude of σ_{ε_q}.

For low values of the standard deviation, the forecasting is effective, while high values yield too broad probability interval for the occurrence of the temperature forecasted. Nevertheless, the computations carried out determined the reference level for the introduction of more complex forecasting methods the application of which might be justified by comparison with the AR(1) results. In winter months when the minimum temperature of the water is $-0.5°C$, the AR(1) assumptions can be used approximately under the condition:

$$x_q(t_j) - \sigma_{x_{qp}} \geqslant -0.5°C,$$

where $\sigma_{x_{qp}}$ is the value of σ_{x_q} assumed from $N(0,\sigma_{\varepsilon_q})$ intervals.

The problem of the ice forecast in the winter season needs a special consideration and has particular formulae. The AR (1) parameters estimation was verified with good results on the independent measurements from years 1958–1960.

4. Computations of temperature range in summer months by the Monte Carlo method

The temperature range in individual summer months is one of the essential characteristics of recreative and ecological significance. Respective computations in this field were carried out for June, July, August, and September ($q = 1, 2, 3, 4$) for the three measuring stations considered in the paper. The temperature range can be expressed by the equation:

$$Z_{Nq} = \max x_{qi} - \min x_{qi}, \qquad 0 \leqslant i \leqslant N_q. \tag{8}$$

The determination of an empirical distribution function based on the observations from 35 years is not authoritative due to the statistical properties of equation (8) and a relatively short observation time. This problem was solved applying the simulation by the Monte Carlo method. In such approach the problem resolves into the computation of the distribution functions:

$$\Pr \{Z_{Nq} < Z. \tag{9}$$

A random variable Z_{Nq} is the range of sequence Nq of dependent random variables determined by equation (7), the value of x_{qo} can be found based on the assumption as that randomly selected from the normal distribution of sequence $\{X_{qi}\}$. If there exists a density probability function of joint distribution of variables $\{X_{qi}\}$ determined as $f(x_{q0}, x_{q1}, \ldots, x_{qNq})$, then the distribution function of range (9) meets the equation known in statistics:

$$F_q(z) = \sum_{i=0}^{Nq} \left\{ \int_{-z}^{0} \left[\int_{x_{qi}}^{x_{qi}+z}, \ldots, \int_{x_{qi}}^{x_{qi}+z} f(x_{q0}, x_{q1}, \ldots, x_{qNq}) \prod_{\substack{j=0 \\ j\neq i}}^{Nq} \mathrm{d}x_{qj} \right] \mathrm{d}x_{qi} \right.$$

$$\left. + \int_{0}^{z} \left[\int_{x_{qi}}^{z}, \ldots, \int_{x_{qi}}^{z} f(x_{q0}, x_{qi}, \ldots, x_{qNq}) \prod_{\substack{j=0 \\ j\neq i}}^{Nq} \mathrm{d}x_{qj} \right] \mathrm{d}x_{qi} \right\}. \tag{10}$$

The distribution density function is, according to formula (10), n-dimensional function of the normal distribution:

$$f(x_{q0}, x_{q1}, \ldots, x_{qNq}) = \frac{1}{(2\pi)^{N_q/2}\,\sigma^{N_q}} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^{N_q} (x_{qi} - x_{qi-1})^2 \right]. \tag{11}$$

The computation of the distribution function requires formula (10) to be integrated with the density function being determined according to (11). The problem is difficult to solve and involves the computation of multiple integrals from multi-dimensional normal distribution of dependent random variables. The simulation by the Monte Carlo method was applied according to a scheme published on a similar problem (Kupść, 1969). For each sequence $\{X_{qi}\}$ determined by formula (7), $n = 1000$ realizations $\{X_{qil}\}$ were performed, and for each of them the following value was determined:

$$\begin{gathered} z_1 = \max x_{qil} - \min x_{qil}; \qquad l = 1, 2, \ldots, n, \\ 0 \leqslant i \leqslant N_q. \end{gathered} \tag{12}$$

Mean values $\bar{x}_q$ were assumed as those from monthly mean values from the years 1950–1984 and parameters of AR(1) were taken according to Table 2. The distribution function $\hat{F}_q(z)$ was estimated using the expression:

$$P(A) = \hat{F}_q(z) = \frac{m}{n}, \tag{13}$$

where A is an event with a value of z_{ql} being lower than z assumed, and m is the number of the event realizations in n experiments. Since the realizations of sequences $\{X_{qi}\}$ are mutually independent according to Bernoulli's law of large numbers, the following equation holds true for arbitraty $\varepsilon > 0$:

$$\lim_{n\to\infty} \Pr\left\{ \left| \frac{m}{n} - F(z) \right| < \varepsilon \right\} = 1. \tag{14}$$

Therefore, for $n = 1000$ the determination of the distribution function by the use of formula (13) is a good estimator of the function represented by formula (9). With the magnitude of n being assumed, the distribution of estimator $\hat{F}_q(z)$, determined as $N\{E[F(z)], \sigma_F\}$, is also known, where the value of $\sigma_{\hat{F}}$ can be found from:

$$\sigma_{\hat{F}} = \sqrt{\frac{\bar{F}(z)\,[1-\bar{F}(z)]}{n}}. \tag{15}$$

Having the above in mind the confidence interval of $\hat{F}(z)$ is determined according to:

$$\Pr\{\hat{F}(z) - t_\alpha \sigma_{\hat{F}} \leqslant \bar{F}(z) \leqslant \hat{F}(z) + t_\alpha \sigma_{\hat{F}}\} = 1-\alpha \tag{16}$$

where t_α is such a value of a variable with distribution $N(0, 1)$ that $P(|T| < t_\alpha) = 1-\alpha$. As a result of computations a number of distribution functions and characteristics were obtained illustrated for Międzyzdroje in Figure 4 and for all measuring stations in Tables 3 and 4.

Table 3. Distribution functions of the temperature range in summer months at Międzyzdroje, Mielno, and Władysławowo (according to the computations by the Monte Carlo method)

z [°C]	Międzyzdroje				Mielno				Władysławowo			
	months				months				months			
	VI	VII	VIII	IX	VI	VII	VIII	IX	VI	VII	VIII	IX
1	0	0	0	0	0	0	0	0	0	0	0	0
2	.001	.002	.004	.005	0	0	0	0	0	0	0	0
3	.007	.063	.115	.103	0	.008	0	.005	.003	.016	.006	.028
4	.067	.304	.444	.413	.002	.069	0	.038	.037	.124	.051	.164
5	.269	.632	.787	.719	.012	.301	.020	.177	.165	.378	.205	.419
6	.529	.855	.937	.907	.059	.598	.080	.415	.383	.673	.420	.669
7	.758	.961	.984	.983	.182	.814	.239	.633	.622	.862	.638	.859
8	.900	.996	.996	.997	.372	.940	.440	.814	.817	.962	.794	.934
9	.968	1.0	.999	.998	.570	.988	.649	.933	.904	.993	.913	.972
10	.992		1.0	.999	.726	.996	.795	.977	.958	.999	.965	.990
11	.998			1.0	.859	1.0	.903	.990	.987	1.0	.983	1.0
12	.999				.939		.963	.999	.997		.994	
13	1.0				.971		.986	1.0	1.0		.997	
14					.987		.984				.999	
15					.994		.999				1.0	
16					.997		1.0					
17					1.0							

The data in Table 3 contain the empirical distribution functions of the temperature range at measuring stations under consideration. In Table 4 the means and standard deviations of the temperature range at those stations are presented. Detailed analysis of those data can be elaborated for the local studies of sea water temperature variations or for the description of the near-shore water environment.

It should be emphasized that the simulation by the Monte Carlo method was accomplished on the basis of mean monthly sea water temperatures

Table 4. Mean values and standard deviations of the sea water temperature range at Międzyzdroje, Mielno, and Władysławowo (according to the computations by the Monte Carlo method)

Station	Months							
	VI		VII		VIII		IX	
	$\bar{z}$ [°C]	σ_z [°C]	$\bar{z}$ [°C]	σ_z [°C]	$\bar{z}$ [°C]	σ_z [°C]	$\bar{z}$ [°C]	σ_z [°C]
Międzyzdroje	6.5	1.5	5.2	1.2	4.7	1.1	4.9	1.2
Mielno	9.3	2.0	6.3	1.3	8.9	1.9	7.0	1.6
Władysławowo	7.1	1.7	6.0	1.3	7.0	1.8	6.0	1.5

measured during long time (as for the determination of this parameter) while the parameters of process AR(1) were found from 10 N_q values for a given month. Therefore, the realization series assumed in this case were also reliable. Thus, the data assumed replaced 35 empirical values of distribution functions $\hat{F}_q(z)$ which were possible to obtain from the measurements as differences between the extremum values, the reliable determination of which requires a long observation period.

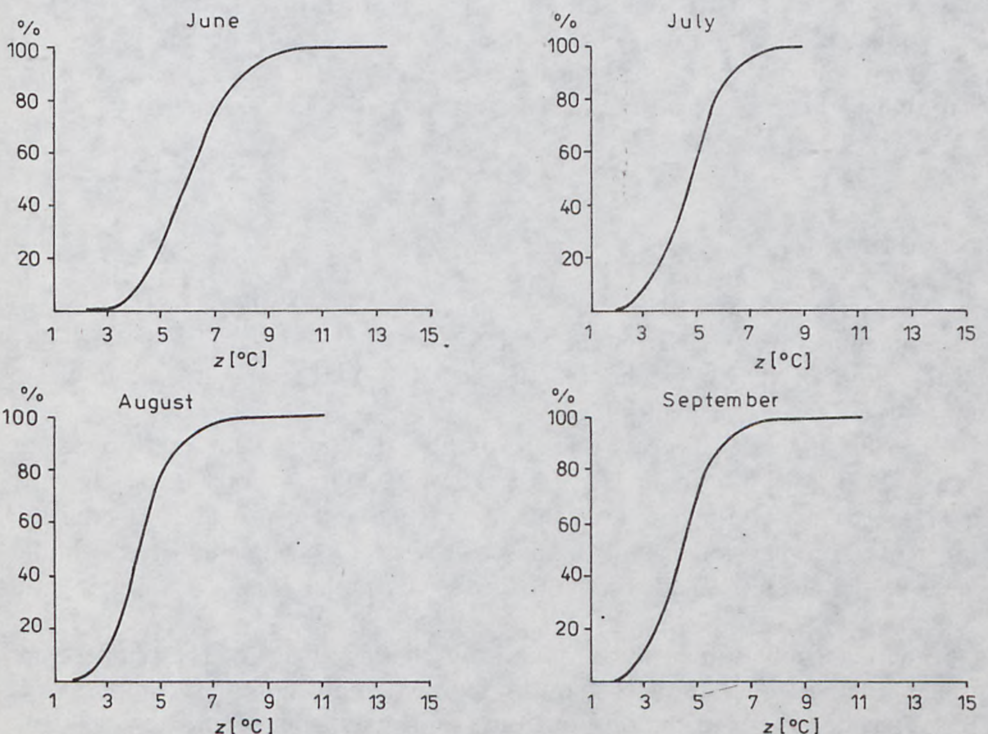

Fig. 4. Distribution function of the sea water temperature range during summer months for Międzyzdroje as computed by the Monte Carlo method

An obvious problem arises whether the simulation applied was consistent with complex physical conditions assumed for the process under consideration. In order to verify the validity of the results of the simulation model assumed, extremum values x_{qi} for each month and for each measuring station were compared with accessible experimental data. It should be emphasized that the simulation did not aim at presenting the distribution of the extremum values, which in this case would require the 'tail' distribution to be analysed. The comparative data demonstrate a good choice of the simulation model. This agreement was verified on the basis of such sensitive indicators as the extremum values which in this case remained within acceptable limits as concluded on the basis of observations.

Similarly as with the autoregressive computations, the largest scatter of the temperature range was found at Mielno, which was due to the location of the measuring station. However, the temperatures in summer are most stable at Międzyzdroje and Władysławowo. Characteristics shown in Tables 3 and 4 determine the stability of temperatures during summer season. The physical aspect of this phenomenon is complex since the coastal waters are warmer as compared to those in the depth and on the surface of the open sea. Rapid mixing of water masses of different temperatures may bring about a sudden drop in the coastal water temperature, *eg* have been known the cases of decrease in the coastal water temperature in August to 7–8°C.

Statistical presentation of the temperature range problem is justified having in mind the practical purposes as well as the determination of the variability range of basic physical parameters of the sea water.

5. Summary

The paper presents the computations of the periodic structure of the phenomenon investigated on the basis of which the type of the probability distribution of the daily changes of the sea water temperature and mean monthly values were determined.

The usefulness of process AR(1) was also determined for the forecast at 12.00 GMT with a 24-hour lead time. The possibility of forecasting given by a presented method suggests its use in practice and can also be a criterion for more complex forecasting models. The latter – when employed – should yield better results than the autoregressive model which is simple and economic.

The determination of the probabilistic characteristics of the temperature ranges in individual summer months required more complex computations which yielded the distribution functions, simple statistical characteristics, and the formulae for the calculation of the confidence intervals.

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